Hermes

A Computational Tool for Proportional Studies in Design

PARK Hyoung-June[1], ECONOMOU Athanassios[2] and PAPALAMBROS Panos[3]
[1] *College of Architecture and Urban Planning, University of Michigan, USA*
[2] *College of Architecture, Georgia Institute of Technology, USA*
[3] *College of Engineering, University of Michigan, USA*

Keywords: proportion, genetic algorithms, design optimization, Palladio

Abstract: A computational tool for proportional analysis and synthesis in architectural composition is presented. The various components of the software are briefly explained and two case studies, one in analysis and one in synthesis of form are presented. Both studies presented here are drawn from Palladio's second book of architecture.

1 INTRODUCTION

The absence of computational tools for the application of proportional theory in analysis and synthesis in design has been a persistent problem in the field of formal composition in architectural design. Analysis of existing designs still requires an enormous amount of patience and persistence from the researcher to undertake with pencil and paper. Synthesis of new designs with proportional qualities is even more elusive because of the mathematical sophistication it demands from the designers. And both activities require a command of this body of knowledge that very few architects nowadays possess.

In this paper a computational tool, Hermes, is introduced as a tool for proportional studies based on the theory of means. The analysis component of the application evaluates existing designs and provides statistical measures about the proportional structure of the design. The synthesis component of the application generates new designs from known ones with additional prescribed proportional properties. In both cases design optimization methodologies are employed (Papalambros and Wilde, 2000). The analysis component is written in Autolisp and runs within AutoCAD. The synthesis component is written using the Genetic Algorithm Toolbox of Matlab and an Autolisp application within AutoCAD.

In order to link this project to the long list of research projects that have tackled proportion, the corpus of the designs for both analysis and synthesis has been selected from Palladio's buildings (Mitrovic 1990; Hersey and Freedman 1992; March 1998; Howard and Longair 1982; Tavernor and Schofield 1997).

2 RATIO, PROPORTION AND PROPORTIONALITY

A *ratio* is a relation between two numbers and *proportion* is a relation between two ratios. The least set of numbers that can establish a proportion is 3. For 3 numbers x, y, z, and $x<y<z$, there are 3 possible outcomes of comparisons, 1 unique case of equality, $x:y = y:z$, and 2 cases of inequality, $x:y<y:z$ and $x:y>y:z$. For each case of inequality, there can be an infinite number of subcases with respect to the actual numbers involved in the comparison. Among these relationships, some are more significant than others; for example, for three numbers x, y, z, and $x<y<z$, if $(1/z)-(1/y)=(1/y)-(1/x)$, the inequality $x:y<y:z$ can be rewritten as an equality, namely, $(z-y)/z=(y-x)/x$. The problem has been nicely solved in antiquity by Greek mathematicians in a series of successive attempts, initially proposing 2 more such equalities by Archytas, the arithmetic and the harmonic mean, latter 3 more, possibly by Eudoxus, and finally 2 additional distinct sets of 4 by Nicomachus and Pappus respectively, with 3 overlapping cases among them, bringing the total number of inequalities to 10. These 10 relationships of ratios plus the first initial relation of equality, the geometric mean, brought the number of comparisons to 11 and they were all treated uniformly under the heading of *proportionality* or *theory of means* (Heath 1921). Among these 11 ways of comparing 2 ratios involving 3 numbers, only 4 survive in current discourse, the initial 3, the geometric, arithmetic, and harmonic mean, and the extreme and mean ratio, otherwise currently known as the golden mean (Herz-Fischler 1998). The defining relationships for all 11 means and the corresponding definitions for each of the terms with respect to the two other terms of the proportion are given in Figure 1.

P_1:	$\frac{z-y}{y-x}=\frac{x}{x}$,	$x=2y-z$	$y=\frac{x+z}{2}$	$z=-x+2y$
P_2:	$\frac{z-y}{y-x}=\frac{z}{y}$,	$x=\frac{y^2}{z}$	$y=\sqrt{xz}$	$z=\frac{y^2}{x}$
P_3:	$\frac{z-y}{y-x}=\frac{z}{x}$,	$x=\frac{yz}{2z-y}$	$y=\frac{2xz}{x+z}$	$z=\frac{xy}{2x-y}$
P_4:	$\frac{z-y}{y-x}=\frac{x}{z}$,	$x=\frac{y\pm\sqrt{y^2-4(z^2-yz)}}{2}$	$y=\frac{x^2+z^2}{x+z}$	$z=\frac{y+\sqrt{y^2-4(x^2-xy)}}{2}$
P_5:	$\frac{z-y}{y-x}=\frac{x}{y}$,	$x=\frac{y\pm\sqrt{y(5y-4z)}}{2}$	$y=\frac{-x+z+\sqrt{4x^2+(-x+z)^2}}{2}$	$z=\frac{-x^2+xy+y^2}{y}$
P_6:	$\frac{z-y}{y-x}=\frac{y}{z}$,	$x=\frac{y^2-z(z-y)}{y}$	$y=\frac{x-z+\sqrt{4z^2+(-x+z)^2}}{2}$	$z=\frac{y+\sqrt{y^2-4(xy-y^2)}}{2}$
P_7:	$\frac{z-x}{y-x}=\frac{z}{x}$,	$x=z-\sqrt{z^2-yz}$	$y=\frac{2xz-x^2}{z}$	$z=\frac{x^2}{2x-y}$
P_8:	$\frac{z-x}{z-y}=\frac{z}{x}$,	$x=\frac{z\pm\sqrt{z(4y-3z)}}{2}$	$y=\frac{x^2-xz+z^2}{z}$	$z=\frac{x+y+\sqrt{(y-x)(y+3x)}}{2}$
P_9:	$\frac{z-x}{y-x}=\frac{y}{x}$,	$x=\frac{y+z-\sqrt{(y+z)^2-4y^2}}{2}$	$y=\frac{x+\sqrt{x(4z-3x)}}{x}$	$z=\frac{x^2-xy+y^2}{x}$
P_{10}:	$\frac{z-x}{z-y}=\frac{y}{x}$,	$x=z-y$	$y=z-x$	$z=x+y$
P_{11}:	$\frac{z-x}{z-y}=\frac{z}{y}$,	$x=\frac{(2y-z)z}{y}$	$y=\frac{z^2}{2z-x}$	$z=y+\sqrt{y^2-xy}$

Figure 1 The 11 proportionalities

3 COMPUTING PROPORTIONALITIES

The computation of means is a straightforward task. The computation can be, and has been, invaluable so much in analysis or in synthesis in design systems. Still the ways that these numbers interact with one another is not just a mathematical problem but an aesthetic problem. For example, in classical architectural theory proportional triplets typically refer to dimensions of rooms. In classical music theory proportional triplets typically refer to tunings of strings (see, for example, Jeans 1968). The meaning of these triplets, whether in 3-dimensional, 2-dimensional or 1-dimensional design worlds, requires an aesthetic world making.

3.1 Aesthetic Measures

Here four very basic measures are formed; The Individual Proportionality Value ($\boldsymbol{P}_k$) for $k\leq 11$, is defined as the percentage of the number of triplets of dimensions in a design that belongs to a specific proportionality. The Proportionality Value ($\boldsymbol{P}$) is defined as the sum of all $\boldsymbol{P}_k$ for $k\leq 11$ and signifies the percentage of all triplets of dimensions that belongs to any proportionality. The Individual Proportionality Value Remnant ($\boldsymbol{R}_k$) for $k\leq 11$, is the inverse of the Individual Proportionality Value ($\boldsymbol{P}_k$) for $k\leq 11$, and refers to the percentage of triplets of sets of dimensions that do not belong to a specific proportionality. The Remnant Value ($\boldsymbol{R}$) is the inverse of the $\boldsymbol{P}$ and signifies the percentage of all triplets of dimensions that do not belong to any proportionality. The computation of these four aesthetic measures is shown in Figure 2. The entries in the computation are: N the number of dimensions in a design, T the number of triplet combinations from N, a_k the algorithm checking T for proportionality k, $k\leq 11$, P_k the number of triplet combinations satisfying a proportionality k, $k\leq 11$, P the number of triplet combinations satisfying any proportionality requirement, R_k the number of triplet combinations not satisfying a proportionality k, $k\leq 11$, and R the number of triplet combinations not satisfying any proportionality requirement.

$$T=(N,3)=\frac{N!}{(3!(N-3)!)}$$

$$P_k=a_kT$$

$$P=\sum_{k=1}^{11}P_k$$

$$T\leq\sum_{k=1}^{11}P_k+R$$

$$\boldsymbol{P_k}=\frac{100\times P_k}{T}$$

$$\boldsymbol{P}=\sum_{k=1}^{11}\boldsymbol{P_k}$$

$$\boldsymbol{R}=100-\boldsymbol{P}$$

Figure 2 Computation of proportionality measures

The aesthetic measures of the Individual Proportionality Value ($\boldsymbol{P_k}$), the Proportionality Value ($\boldsymbol{P}$) and their inverses ($\boldsymbol{R_k}$) and ($\boldsymbol{R}$), can be used in a straightforward way in the analysis of existing designs or the design of new ones. For any design, and any set of dimensions associated with that, it is a straightforward task to identify all triplets of dimensions that have proportionality properties and check their relations over the whole set of dimensions. Note however that these proportional measures do not provide the intuitive measurements of local proportional relationships in definitive parts of a design, say, rooms. For the identification of such measures more careful modeling is needed; here the emphasis is given to the design of a seamless automated system that can compute proportions in design and future work will incorporate finer aesthetic systems as well as more complex statistical models, such as factor analysis, Eigenvalue decompositions and so forth.

3.2 Analysis

In analysis, the task is to check whether certain triplets of dimensions given by the architect or simply found by measurements, do in fact yield proportionality relationships. This task is tackled in the analysis component of Hermes, a program written in Autolisp and running within AutoCAD. The input can either be a list of numbers or a list of shapes whose dimensions can be extracted from their geometry within AutoCAD. Hermes computes every possible triplet combination within this set of numbers and generates a spreadsheet with the results of the computation. The structure of the analysis component of Hermes is shown in Figure 3.

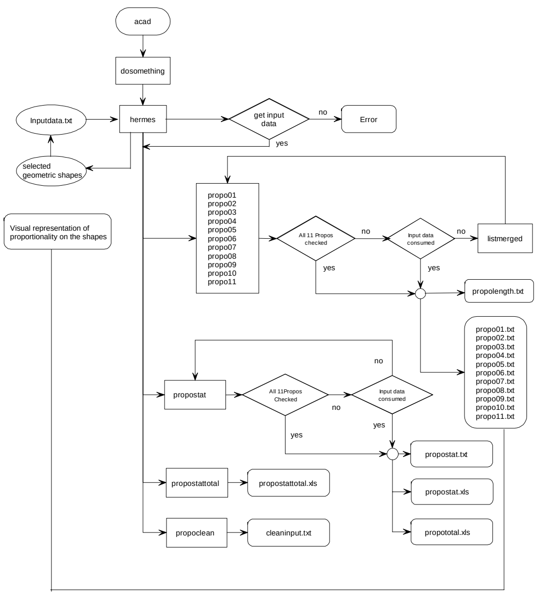


Figure 3 The structure of the analysis component of Hermes

The output of this analysis is given in two different formats that nicely complement the two types of descriptions of the input; a list of numbers with the various proportional measures or a list of weighted shapes that show visually the correspondences between the various proportionalities one against the other in different coloured lines.

3.3 Synthesis

In synthesis, the task is to find for any pair of dimensions some other dimensions that are related to both of them in a proportional relationship; this dimension could be either a lower, medium, or an upper term in the tripartite relationship. This task is tackled in the synthesis component of Hermes, an assortment of applications, one written using the genetic algorithm toolbox of Matlab and another written in Autolisp and running within AutoCAD. The algorithm for the combinatorial search is based on genetic algorithms (GA) that compute smart enumerations imitating natural evolution processes (Goldberg 1989). The results of the combinatorial search are loaded in a custom made Autolisp program within AutoCAD to generate two-dimensional and three-dimensional visual representations from the output of the optimization.

In general four basic constraints are considered here necessary for the shape computation in the synthesis component: overlap, adjacency, shape, and alignment constraints. Overlap constraints ensure design components do not occupy the same area. Adjacency constraints keep the design components linked closely together. Shape constraints ensure every design component keeps its geometry under a Euclidean transformation. Alignment constraints ensure the correct alignment of shapes one to another after their transformation. Additionally, sets of dimensions of the design are taken as design variables which are optimized to maximize or minimize accordingly the proportionality value ***P*** and the remnant value ***R*** of the design problem. The structure of the synthesis component is shown in Figure 4.

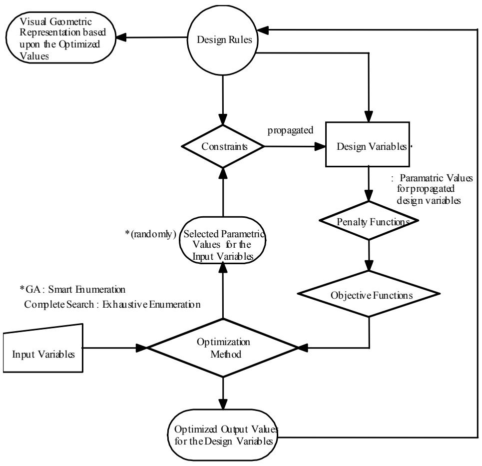


Figure 4 The structure of the synthesis component of Hermes

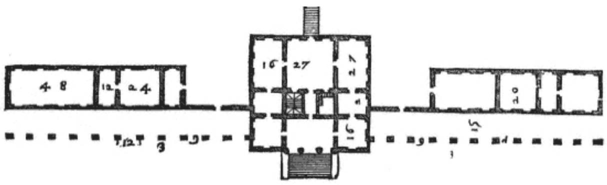
The output of the synthesis is given in two different output formats that nicely parallel the analysis output; a list of numbers with the various proportional measures and a visualization of the proposed optimized design solution. Additionally two more types of spatial descriptions, an image and an animation, are provided to facilitate the bookkeeping of the whole process; the image is a screen capture of the input settings, and the video is an animation made of the series of visual representations of the best fit designs for every generation.

4 PALLADIO REVISITED

Two studies are undertaken here to test the analysis and the synthesis components of Hermes; both of them deal directly with Palladio's work to link the research conducted here to the long list of research projects that have tackled proportion. For the case study in analysis, the corpus of the designs is 38 buildings from Palladio's work. For the case study in synthesis, the corpus is the infamous Villa Rotonda.

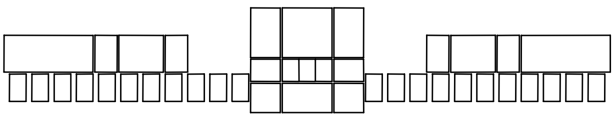
4.1 Analysis Input

The corpus of designs for this case study consists of the complete set of buildings that Palladio included in his Second Book on Architecture (Tavernor and Schofield 1997) except 4 antique reconstructions. Most of these 38 buildings were illustrated in the original publication with a single plan, a single section, few numbers, and a brief text; there is no doubt that this economy of description has given numbers an deeper sense of meaning in Palladio's work (March 1998). This corpus is illustrated here in terms of two representations; the first is a straightforward numerical description, a list of numbers associated with each design. The second is a pictorial description, an abstract plan designed in AutoCAD consisting of simple geometrical shapes based on the numbers of the original plan. Figure 5 shows a typical input for proportionality analysis.



a)

b) (9 12 15 16 20 24 27 48)



c)

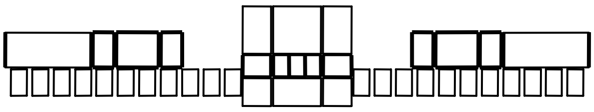
Figure 5 Villa Emo. a) Original plan.
b) Numerical input c) Shape input: Abstract plan

4.2 Analysis Output

Any query about proportionality measures in the Palladian corpus, at least in the way these measures are specified in this paper, may provide interesting and quite often unexpected results. Table 1 shows a comprehensive summary of these aesthetic measures for the entire Palladian corpus. The Palladian corpus has been sorted here according to programmatic features: The list starts with the 9 palaces from Palace Antonini to Palace Barbarano, continues with the 9 Villas for Venetian Patrons from Villa Bagnolo to Villa Emo, continues with the 12 villas from the Villa Saraceno to Villa Sarego at Miega, continues with one antique villa reconstruction, and ends up with 7 inventions for various sites. Other sortings are obviously possible and in fact desirable. In Table 1, *N* is the number of dimensions given in the original plan, *T* is the number of possible triplets that can be formed from the initial set of dimensions, *P* is the number of triplets that have some proportionality value, and ***P*** is the proportionality value for each villa as defined earlier in this paper.

The data suggest a variety of different insights on the proportional structure of the Palladian corpus. It is clear that few of the villas evince a clear affinity to the theory of means. Still some of the villas exhibit a high percentage of proportionality values as defined in this computation, others none, and still not all proportionality values have the same weight in the distribution of the corpus. Interestingly enough the plan of the Vila Sarego at Santa Sofia collects the greatest proportionality value of 60% followed by the Villa Thiene at Cicogna with 50%, followed by the Palace Angarano with 50% while other designs such as the Palace della Torre or the Palace Capra are the last in the list with no proportionality value defined, at least in terms of the theory of means. Equally interesting, not all 11 proportionalities feature in the corpus with the same frequency; as perhaps it is expected, P_1, the arithmetic mean, P_3, the harmonic mean, and P_{10}, the extreme and mean ratio or otherwise known nowadays the golden section, are more prominent and feature respectively at 25%, 11% and 21%; what is interesting though is that the proportionality P_7, a mean that never really enjoyed a proper name in antiquity or onwards, is prominent in the table with 14%. Perhaps more surprising is the fact that the only true proportion, that is the proportion that involves direct equality of ratios, the geometric mean, features rather low with 8%.

But Hermes can do more than computation of numbers. A nice feature of the analysis component is that the system automatically generates pictorial representations of the proportionality relationships as these are found in the designs. Figure 6 shows one of the diagrams that are generated automatically illustrating the proportionalities in Villa Emo; the specific diagram picks up all the parts of the plan that are related through the P_4 mean.



$P_4 =$ (12 20 24)

Figure 6 Villa Emo. Diagrammatic representations of P_4

Table 1 Proportional measures in Palladio's corpus

	N	*T*	*P*	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P
Palace Antonini	9	84	6	3.57	0	0	0	0	0	0	0	0	3.57	0	7.14
Palace Chiericati	9	84	8	3.57	0	0	0	1.19	0	2.38	1.19	0	1.19	0	9.52
Palace Porti	5	10	3	10	0	0	0	0	0	0	0	10	10	0	30
Palace della Torre	11	165	21	4.84	1.21	1.81	0	0	0	1.21	0.61	0	2.42	0.61	12.72
Palace Thiene	8			0	0	0	0	0	0	0	0	0	0	0	0
Palace Valmarana	15			1.31	0	0.21	0	0.22	0	0.44	0.22	0.22	0.66	0.22	3.52
Villa Rotonda	6	20	3	0	0	0	5	5	0	0	0	0	5	0	15
Palace Capra	5	10	0	0	0	0	0	0	0	0	0	0	0	0	0
Palace Barbarano	11	165	14	3.03	0.61	0.61	0	0	0	0	0	0	3.63	0.61	8.48
Villa Bagnolo	9	84	13	4.76	2.38	0	0	0	0	1.19	1.19	1.19	3.57	1.19	15.47
Villa Badoer	9	84	16	8.33	1.19	0	0	1.19	0	2.38	1.19	1.19	3.57	0	19.04
Villa Zeno	5	10	1	10	0	0	0	0	0	0	0	0	0	0	10
Villa Foscari	7	35	6	2.85	2.85	2.85	0	0	0	2.85	2.85	0	0	2.85	17.14
Villa Babaro	12	220	51	7.72	1.36	1.81	0.45	0.45	0	2.72	1.81	0.91	4.54	1.36	23.18
Villa Pisani,M	6	20	1	0	0	0	0	0	0	0	0	0	5	0	5
Villa Cornaro	7	35	3	2.85	0	0	0	0	0	2.85	2.85	0	0	0	8.57
Villa Mocenigo	5	10	2	10	0	0	0	0	0	0	0	0	10	0	20
Villa Emo	8	56	15	5.35	3.57	5.35	1.78	0	0	3.57	0	0	3.57	3.57	26.78
Villa Saraceno	6	20	2	0	0	0	0	0	0	5	0	0	5	0	10
Villa Ragona	6	20	1	5	0	0	0	0	0	0	0	0	0	0	5
Villa Poiana	8	56	12	3.57	0	3.57	1.78	0	0	1.78	1.78	1.78	3.57	3.57	21.42
Villa Valmarana	9	84	5	2.38	0	0	0	0	0	1.19	0	0	1.19	1.19	5.95
Villa Trissino,M	8	56	14	3.57	1.78	5.35	3.57	0	0	1.78	1.78	1.78	3.57	1.78	24.99
Villa Repeta	6	20	1	0	0	0	0	0	0	0	0	0	5	0	5
Villa Thiene,C	4	4	2	0	0	25	0	0	0	25	0	0	0	0	50
Villa Angarano	8	56	5	3.57	0	0	1.78	0	0	1.78	1.78	0	0	0	8.92
Villa Thiene, Q	15	455	7	0.87	0	0	0	0	0	0	0	0	0.65	0	1.53
Villa Godi	12	220	29	4.09	0.90	0.91	0.45	0	0	1.81	0.45	0.91	2.72	0.91	13.18
Villa Sarego, S	6	20	12	10	0	10	5	0	5	10	5	5	5	5	60
Villa Sarego, M	11	165	34	5.45	1.21	3.03	1.21	0.61	0	2.42	1.21	0.60	3.03	1.81	20.61
Villa of Ancients	12	220	13	2.72	0	0.45	0	0	0.45	0.45	0.45	0	0.91	0.45	5.91
Venetian Palace, A	9	84	7	2.381	1.19	0	0	0	0	1.19	1.19	0	2.38	0	8.33
Venetian Palace, B	12	220	29	3.63	1.36	1.36	0.45	0.45	0.91	1.36	0.45	0.91	2.27	0	13.18
Palace Trissino, V	12	220	16	3.18	1.36	0	0	0	0	0.45	0.45	0	1.36	0.45	7.27
Palace Angarano	4	4	2	0	25	0	0	0	0	0	0	0	25	0	50
Palace della Torre	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0
Palace Garzadore	6	20	6	10	0	0	0	5	0	5	5	0	5	0	30
Villa Mocenigo, B	11	165	20	3.03	0	1.81	0.61	0.61	0	0.61	0.61	1.21	2.42	1.21	12.12

4.3 Synthesis Input

Any design from the Palladian corpus can be used as the initial shape in the synthesis component of Hermes to produce parametric variations based on different proportionality values. Here the design selected for parameterization and variation is the infamous Villa Almerico mostly known as Villa Rotonda. There are several masterfully treatments of the villa design; for recent formal treatments see particularly (Howard and Longair 1980; Mitrovic 1990, 2004; March 1998).

The input for the computation of the parametric variations of the villa in Hermes is similar to the input to the analysis component. This input is as before, numerical and pictorial. The key difference with the input in the analysis component is that the dimensions are considered here variables to be transformed. Furthermore, 2 additional levels of representations are introduced to capture a rising complexity in the delineation of the problem. The first level of representation is just the 2-dimensional plan and the initial variables default to the 6 dimensions of the plan. The variables involved in this computation are 6 and the initial values are (30, 26, 15, 11, 6, 12). The second level of representation refers to a 3-dimensional *parti* model basically meshing the numbers and dimensions from the plan and the section as specified by Palladio. The variables involved in this computation are 12 and the initial values of the 6 additional sectional variables are (55, 21.75, 18, 10, 7, 6.5). The third level of representation refers to a more complex geometry where several aspects of the models are parameterized to account for all design variables that can be deduced from the original setting. The variables involved in this computation are 32. Figure 7 shows these 3 different versions of the design.

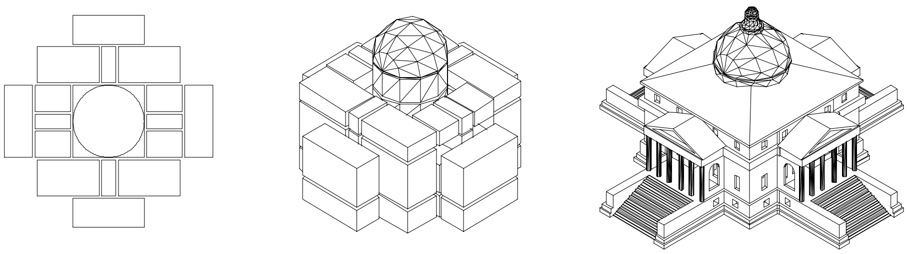


Figure 7 Parametric representations of the Villa Rotonda in Hermes (a) Plan Model (b) Mass Model (c) Detailed Model

4.4 Synthesis Output

The smart enumeration of this optimization problem of finding best proportional variations of the Villa Rotonda is done by using directly the MATLAB Genetic Algorithm Toolbox as well as a series of additional functions programmed within MATLAB. In the first run of the study some 2,254,714 cases were checked. Figure 8 shows three of the designs computed by the Genetic Algorithm component and subsequently automatically generated within the Autolisp program in AutoCAD.

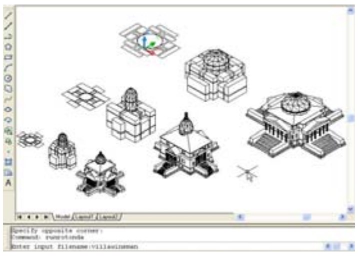


Figure 8 Parametric variations of the Villa Rotonda in Hermes

5 CONCLUSION

A computational tool for proportional studies in architectural composition was presented and two case studies illustrated some of the capabilities of the software. Future work will deal with the development of more refined proportional measures, extensions to more complex optimization models for the extraction of proportional measures, and streamlining of computational procedures used in both analysis and synthesis.

REFERENCES

Goldberg, D.E. 1989. *Genetic Algorithm in Search Optimization and Machine Learning*. Boston: Addison-Wesley.

Heath, T. 1921. *A History of Greek Mathematics*. Oxford: Clarendon Press.

Hersey, G., and R. Freedman. 1992. *Possible Palladian Villas (Plus a Few Instructively Impossible Ones*). Cambridge: MIT Press.

Herz-Fischler, R. 1998. *A Mathematical History of the Golden Number*. NY: Dover.

Howard, D., and M. Longair. 1982. Harmonic Proportion and Palladio's Quattro Libri. *Journal of the Society of Architectural Historians* 41: 116-143. Boston: GSD.

Jean, J. 1968. *Science and Music*. NY: Dover.

March, L. 1998. *Architectonics of Humanism: Essays on Number in Architecture*. London: Academy Editions and John Wiley.

Mitrovic, B. 1990. Palladio's Theory of Proportion. *Journal of the Society of Architectural Historians* 49: 279-292. Boston: GSD.

Papalambros, P.Y., and D.J. Wilde. 2000. *Principles of Optimal Design, Modeling and Computation*. New York: Cambridge University Press.

Tavernor, R., and R. Schofield. 1997. *Andrea Palladio: Four Books on Architecture*. Cambridge: MIT press.