Parametric Variations of Palladio's Villa Rotonda

Hyoung-June Park

Parametric Variations of Palladio's Villa Rotonda

Hyoung-June Park

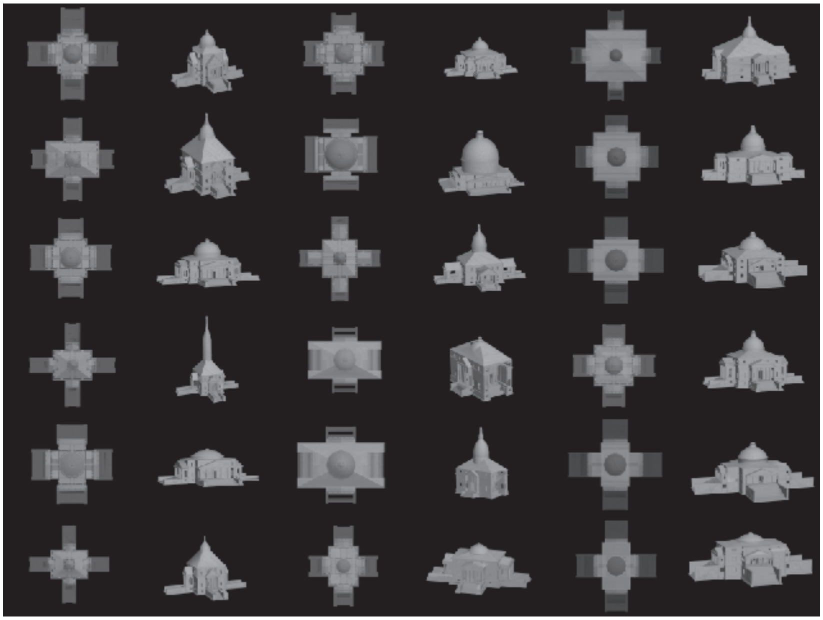
A computational tool for the study of proportional balance is introduced as an apparatus for investigating Andrea Palladio's design of Villa Almerico, more familiarly known as Villa Rotonda, in the second book of his *Quattro Libri dell'Architettura*. The objective of this investigation of Villa Rotonda is to find a novel outcome from the morphological transformations of the villa, where the transformations are generated from parametric variations of the villa while maximizing its proportional balance. The outcome confirms Palladio's mastery of proportional treatments of his design of Villa Rotonda and shows various morphological descendants evolved from the original design. It suggests a new way of employing a parametric geometry in the formal study of a classical building and its stylistic evolution.

1. INTRODUCTION

The concept of morphological transformation with proportional relationships has been developed in art, biology, and architecture throughout the ages (Conway, 1889; Thomson, 1961; Steadman, 1983). D'Arcy Thompson's application of morphological transformations in the process of biological evolution became the impetus for the development of evolutionary modeling in computer science and architecture (Koza, 1992; Bentley, 1999; Broughton and Coates, 1997; Testa et al, 2001). Evolutionary modeling is based upon generation and evaluation. Various design problems have been investigated with the possibility of discovering feasible design solutions. According to given criteria including structure, function, and cost, meaningful and satisfactory design solutions have been produced using a random generative process directed by the probabilistic selection method (Shea and Cagan, 1997; Jagielski et al, 1998; Michalek, 2001). However, the usage of morphological transformation with proportional relationships in the design of artifacts has not been fully developed because it requires hands-on analytical practices in overlaying innumerable geometrical figures on formal schemata with "a proportional divider"(Krier, 1988). The absence of computational tools for the application of proportional theory in analysis and synthesis in design has been a persistent problem in the field of formal composition in architectural design (March, 1998). Analysis of existing designs still requires an enormous amount of patience and persistence on the part of the researcher to undertake with pencil and paper. Synthesis of new designs with proportional qualities is even more elusive because of the mathematical sophistication it demands of designers. In this paper a computational tool called Hermes is implemented based upon the concept of proportionality, a proportional balance achieved by three ordered numbers ($x < y < z$) and their differences ($y - x$, $z - y$, $z - x$). Hermes is employed for analyzing the pattern of proportionality embedded in a given design and synthesizing new designs from morphological transformations of the original design while exploring the parametric variations of its design components. In the process of generating the new designs, design optimization methodologies are employed for tackling combinatorial search problems (Papalambros and Wilde, 2000).

2. RATIO, PROPORTION, AND PROPORTIONALITY

A *ratio* is a relation between two numbers and *proportion* is a relation between two ratios. The least set of numbers that can establish a proportion is three. For three numbers $x, y, z,$ and $x < y < z$, there are three possible outcomes of comparisons, one unique case of equality, $x : y = y : z$, and two cases of inequality, $x : y < y : z$ and $x : y > y : z$. For each case of inequality, there can be an infinite number of sub-cases with respect to the actual magnitudes or multitudes involved in comparison. Among these



◀ Figure 1: Morphological descendents of Palladio's Villa Rotonda.

relationships, some are more important than others. The eleven ways of representing three ordered numbers and their differences with the equalities of the ratios among them were developed in antiquity by ancient Greek mathematicians in a series of successive attempts. They are known as *proportionality* or the *theory of means* (Heath, 1921; March, 1998). These eleven ways of representing proportional balance, the equalities of the ratios among three ordered numbers and their differences, are introduced as an apparatus for generating "commensurability," regarded as a vital component of harmony in design (Morgan, 1960).

$P_1: \frac{z-y}{y-x}=\frac{x}{x},$	$x=2y-z$	$y=\frac{x+z}{2}$	$z=-x+2y$
$P_2: \frac{z-y}{y-x}=\frac{z}{y},$	$x=\frac{y^2}{z}$	$y=\sqrt{xz}$	$z=\frac{y^2}{x}$
$P_3: \frac{z-y}{y-x}=\frac{z}{x},$	$x=\frac{yz}{2z-y}$	$y=\frac{2xz}{x+z}$	$z=\frac{xy}{2x-y}$
$P_4: \frac{z-y}{y-x}=\frac{x}{z},$	$x=\frac{y\pm\sqrt{y^2-4(z^2-yz)}}{2}$	$y=\frac{x^2+z^2}{x+z}$	$z=\frac{y+\sqrt{y^2-4(x^2-xy)}}{2}$
$P_5: \frac{z-y}{y-x}=\frac{x}{y},$	$x=\frac{y\pm\sqrt{y(5y-4z)}}{2}$	$y=\frac{-x+z+\sqrt{4x^2+(-x+z)^2}}{2}$	$z=\frac{-x^2+xy+y^2}{y}$
$P_6: \frac{z-y}{y-x}=\frac{y}{z},$	$x=\frac{y^2-z(z-y)}{y}$	$y=\frac{x-z+\sqrt{4z^2+(-x+z)^2}}{2}$	$z=\frac{y+\sqrt{y^2-4(xy-y^2)}}{2}$
$P_7: \frac{z-x}{y-x}=\frac{z}{x},$	$x=z-\sqrt{z^2-yz}$	$y=\frac{2xz-x^2}{z}$	$z=\frac{x^2}{2x-y}$
$P_8: \frac{z-x}{z-y}=\frac{z}{x},$	$x=\frac{z\pm\sqrt{z(4y-3z)}}{2}$	$y=\frac{x^2-xz+z^2}{z}$	$z=\frac{x+y+\sqrt{(y-x)(y+3x)}}{2}$
$P_9: \frac{z-x}{y-x}=\frac{y}{x},$	$x=\frac{y+z-\sqrt{(y+z)^2-4y^2}}{2}$	$y=\frac{x+\sqrt{x(4z-3x)}}{x}$	$z=\frac{x^2-xy+y^2}{x}$
$P_{10}: \frac{z-x}{z-y}=\frac{y}{x},$	$x=z-y$	$y=z-x$	$z=x+y$
$P_{11}: \frac{z-x}{z-y}=\frac{z}{y},$	$x=\frac{(2y-z)z}{y}$	$y=\frac{z^2}{2z-x}$	$z=y+\sqrt{y^2-xy}$

◀ Figure 2: The eleven proportionalities.

3. COMPUTING PROPORTIONALITIES

The computation of proportionality is a straightforward task for finding the number of three basic measures that have the equality of the ratios among the measures and their differences from given input dimensions. First, the Individual Proportionality Value ($\boldsymbol{P}_k$) for $k \leq 11$, is defined as the percentage of the number of triplets of dimensions in a design that belongs to a specific proportionality. Second, the Proportionality Value ($\boldsymbol{V}_p$) is defined as the sum of all $\boldsymbol{P}_k$ for $k \leq 11$ and signifies the percentage of all triplets of dimensions that belongs to any proportionality. Third, the Remainder Value ($\boldsymbol{V}_r$) is the inverse of the $\boldsymbol{V}_p$ and signifies the percentage of all triplets of dimensions that do not belong to any proportionality. $\boldsymbol{V}_p$ can be greater than 100% because a certain triplet can belong to multiple proportionalities. The computation of proportionality measures is given in Eqn (1).

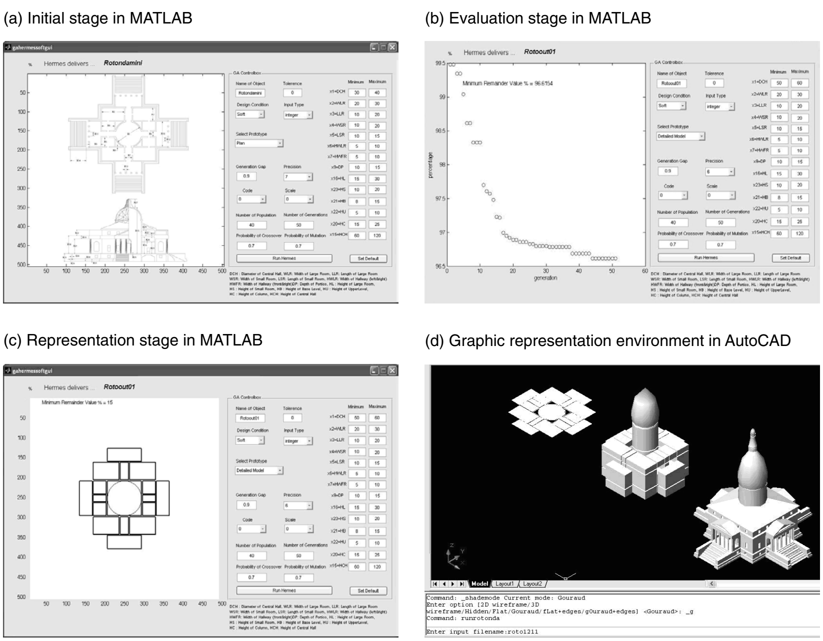
$$\begin{aligned}
\boldsymbol{T} &= (N,3) = \frac{N!}{(3!(N-3)!)} \\
\boldsymbol{L}_k &= \boldsymbol{a}_k(\boldsymbol{L}) \\
\boldsymbol{T}_k &= |L_k| \\
\boldsymbol{P}_k &= \frac{100 \times \boldsymbol{T}_k}{\boldsymbol{T}} \\
\boldsymbol{V}_p &= \sum_{k=1}^{11} \boldsymbol{P}_k \\
\boldsymbol{V}_r &= 100 - \boldsymbol{P}
\end{aligned} \tag{1}$$

The entries in the computation are: $\boldsymbol{N}$ the number of dimensions in a design, $\boldsymbol{T}$ the number of triplet combinations from $\boldsymbol{N}$, $\boldsymbol{L}$ the list of the triplet combinations from the dimensions, $\boldsymbol{a}_k$ the sorting algorithm checking every triplet in $\boldsymbol{L}$ for its fit to the definition of proportionality k, $k \leq 11$, $\boldsymbol{L}_k$ the list of triplet combinations sorted from $\boldsymbol{a}_k$, and $\boldsymbol{T}_k$ the number of triplets in the list $\boldsymbol{L}_k$. The Individual Proportionality Value ($\boldsymbol{P}_k$), the Proportionality Value ($\boldsymbol{V}_p$) and the Remainder Value ($\boldsymbol{V}_r$) are employed as the measures in the analysis of existing designs as well as the synthesis of new design from known ones.

In analysis, the task of Hermes is to compute the proportional measures such as $\boldsymbol{L}_k$, $\boldsymbol{P}_k$, and $\boldsymbol{V}_p$ of input dimensions. The input dimensions are given as a set of numbers directly defined by the user from the user interface of Hermes. The computation yields the pattern of proportional balance laid on the morphological structure of the design. In synthesis, the task of Hermes is to find an optimum design among the morphological transformations derived from parametric variations of a given design while maximizing its Proportionality Value $\boldsymbol{V}_p$, and searching for new dimensions of the given design within the range of input variables. Various algorithms have been employed for controlling the search process and size involved in generating designs. Among them, Genetic Algorithm (GA) was selected for not only finding a global optimum but also for producing additional feasible design solutions to investigate (Michalek, 2001; Kelly et al, 2006). This interactive nature of the algorithm facilitates the generation of morphological descendents of the original villa. Imitating natural evolution processes, GA performs a heuristic search process based upon smart enumerations in order to control a possible combinatorial explosion (Goldberg, 1989; Bentley, 1999; Michalek, 2001). The fitness function *f(r)* for the optimization of Hermes is to minimize the Remainder value $\boldsymbol{V_r}$ while increasing the number of the triplets that belong to any proportionality. When the design variables are $\boldsymbol{D_i}$ and the input variables $\boldsymbol{I_j}$ such that $1 \leq i \leq s$, $1 \leq j \leq t$ where $s \geq t$, *f(r)* is given in Eqn (2).

$$\begin{aligned} f(r) &= V_r \text{ from the dimension of design variables } D_1, \ldots, D_s \\ &= 100 - V_p \\ &= 100 - \sum_{k=1}^{11} P_k \\ &= 100 - \sum_{k=1}^{11} \frac{100 \times T_k}{T} \\ &= 100 - \sum_{k=1}^{11} \frac{100 \times T_k}{C(n,3)} \text{ such that } T_k = (I_1, \ldots, I_t) \end{aligned} \tag{2}$$

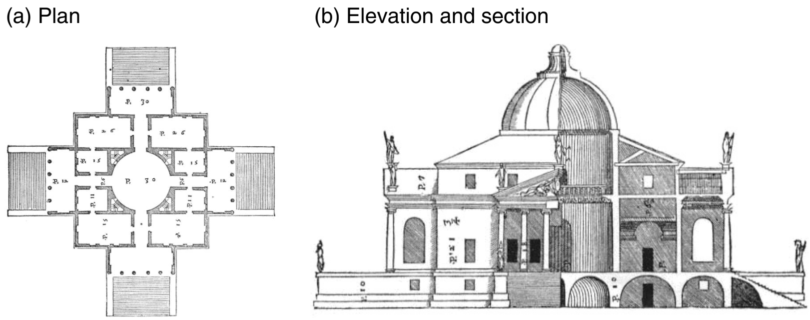
The analysis and synthesis components are written in MATLAB using the Genetic Algorithm Toolbox. The geometric models based upon parametric values computed from MATLAB are generated by AutoLISP application within AutoCAD.



◄ Figure 3: Various stages in MATLAB and AutoCAD.

4. PALLADIO'S VILLA ROTONDA REVISITED

In the second book of his *Quattro Libri dell'Architettura*, Palladio explains the plan, elevation, and section of Villa Rotonda with numbers inscribed on the plate. The numbers represent the dimensions of design components of the villa. The design of Villa Rotonda is a mathematical challenge to Palladio in realizing architecturally irrational numbers and Pythagorean arithmetic. Several treatments of Palladio's design show his method of materializing symbolic meaning into Platonic forms and architectural components with arithmetic computation of the dimensions (Howard and Longair, 1982; Mitrovic, 1990; March, 1998).



► Figure 4: Villa Rotonda in the second book (from Tavernor and Schofield, 1997).

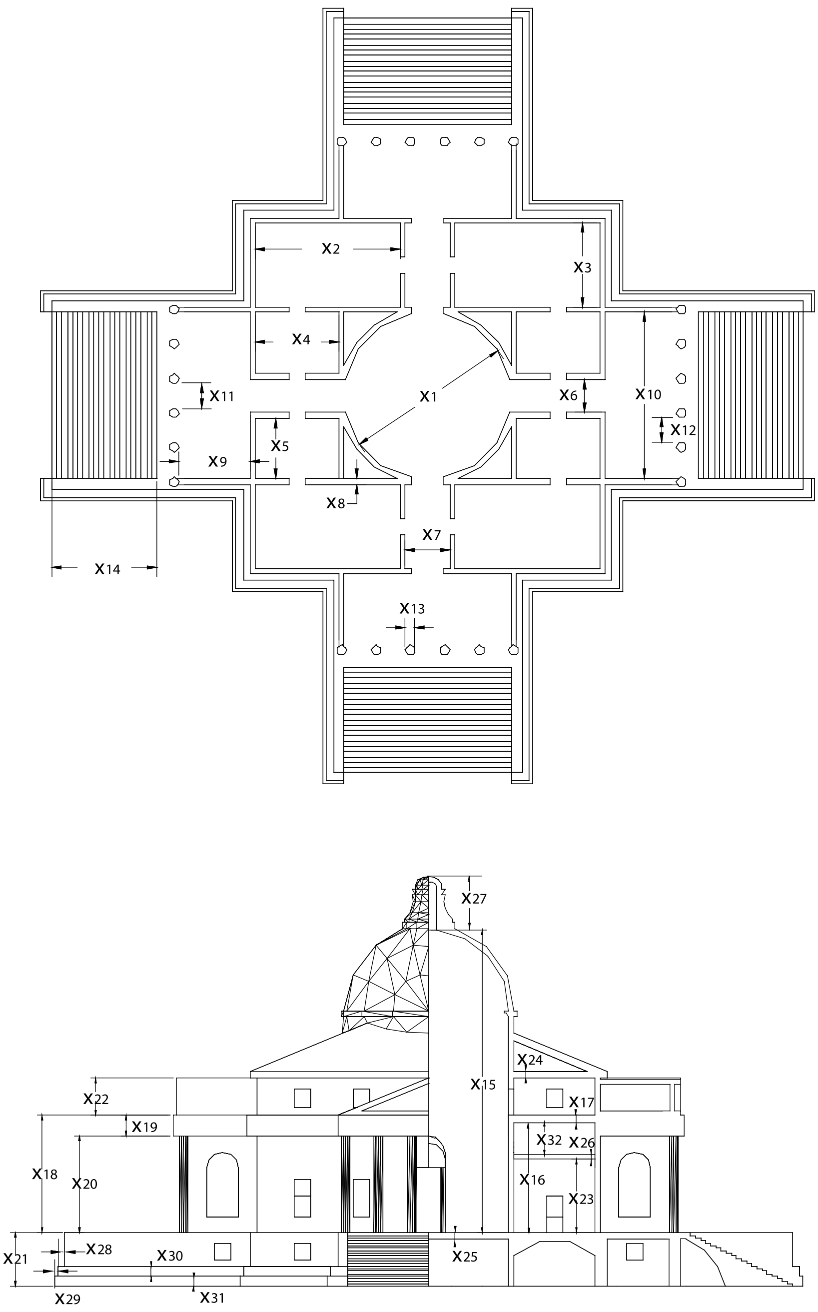
Based upon the treatments, Palladio's design rules for Villa Rotonda are formulated with 38 variables that consist of six different proportionalities and 32 design components, as shown in Figure 5. The original dimensions of the design components and the proportionalities according to Palladio's design of Villa Rotonda are given in Table 1 (March, 1998; Mitrovic, 1990, 2004; Semenzato, 1968).

5. MATHEMATICAL MODELS

With the 38 design variables including six different proportionalities, three different geometric prototypes in plan, mass model, and detailed model were constructed as the prototypes of Villa Rotonda. Mathematical models employed in this research are abstract descriptions of Villa Rotonda using mathematical expressions of morphological structure in three prototypes. Mathematical models employed for this research consist of input variables, constraints, fitness functions, penalty functions, optimization method and output.

5.1. Input variables

Among the 38 design variables, input variables are selected according to different design conditions (hard, medium, and soft) of Villa Rotonda. Design conditions refer to different levels of restriction imposed by the design rules and geometric relations of the original villa. The input variables have their



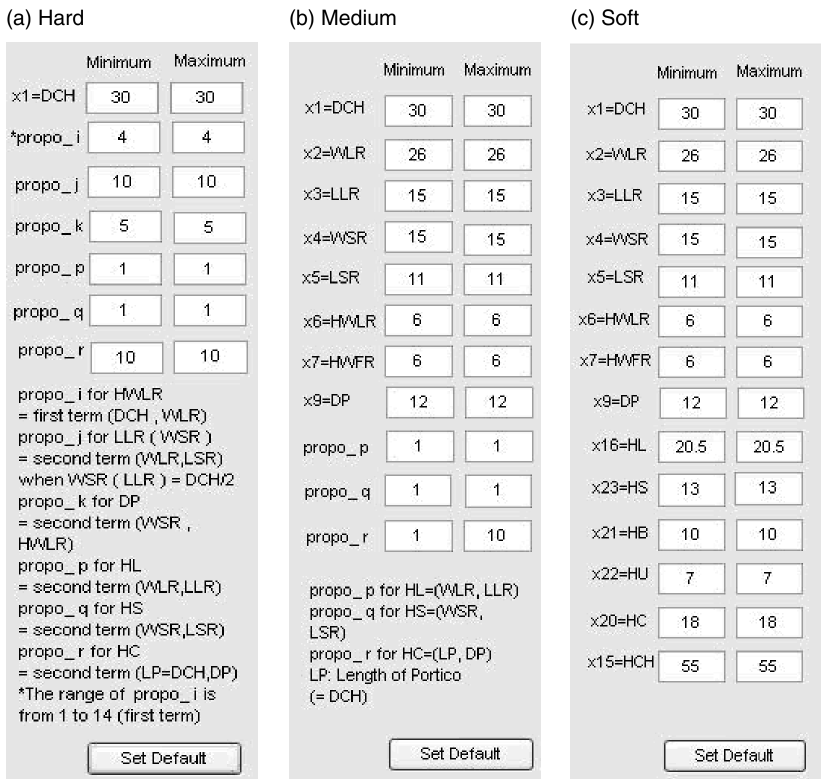
◀ Figure 5: The variables of 32 design components.

minimum and maximum ranges of dimensions. By exploring parametric variations within the ranges of the input variables, a designer finds an "optimum" design artifact that has the best proportionality value. The input variables according to the three design conditions on a user-interface of Hermes are shown in Figure 6.

► **Table 1: The 38 design variables with the original dimensions and numbers of Villa Rotonda.**

Dimension	Component	Design Variable
30	Diameter of Central Hall (*DCH*)	x_1
26	Width of Large Room (*WLR*)	x_2
15	Length of Large Room (*LLR*)	x_3
15	Width of Small Room (*WSR*)	x_4
11	Length of Small Room (*LSR*)	x_5
6	Width of Left and Right Side Hallway (*HWLR*)	x_6
6	Width of Front and Rear Side Hallway (*HWFR*)	x_7
1	Thickness of Wall (*TW*)	x_8
12	Depth of Portico (*DP*)	x_9
30	Length of Portico (*LP*)	x_{10}
5.25	Central Intercolumniation (*CI*)	x_{11}
3.9375	Side Intercolumniation (*SI*)	x_{12}
2	Diameter of Column (*DC*)	x_{13}
22	Length of Stairway (*LS*)	x_{14}
55	Height of Central Hall (*HCH*)	x_{15}
20.5	Height of Large Room (*HL*)	x_{16}
1.25	Thickness of Large Room Ceiling (*TLC*)	x_{17}
21.75	$x_{19} + x_{20}$	x_{18}
3.75	Height of Entablature (*HE*)	x_{19}
18	Height of Column (*HC*)	x_{20}
10	Height of Base Level (*HB*)	x_{21}
7	Height of Upper Level (*HU*)	x_{22}
13	Height of Small Room (*HS*)	x_{23}
1.25	Thickness of Upper Level Ceiling (*TUC*)	x_{24}
1.25	Thickness of Base Level Ceiling (*TBC*)	x_{25}
1	Thickness of Small Room Ceiling (*TSC*)	x_{26}
17.0095	Height of Top Dome (*HTD*)	x_{27}
1.25	Thickness of Inner Tapering (*TIT*)	x_{28}
0.625	Thickness of Outer Tapering (*TOT*)	x_{29}
1.875	Height of Inner Tapering (*HIT*)	x_{30}
1.875	Height of Outer Tapering (*HOT*)	x_{31}
6.5	$x_{16} - (x_{23} + x_{26})$	x_{32}
Number	**Proportionality**	**Design Variable**
4	Proportionality relation among (*HWLR, WLR, DCH*)	*propo_i*
10	Proportionality relation among (*LSR, LLR or WSR, WLR*)	*propo_j*
5	Proportionality relation among (*HWLR, DP, WSR*)	*propo_k*
1	Proportionality relation among (*LLR, HL, WLR*)	*propo_p*
1	Proportionality relation among (*LSR, HS, WSR*)	*propo_q*
10	Proportionality relation among (*DP, HC, LP*)	*propo_r*

Hard, medium, and soft design conditions include different sets of input variables. Descriptions of the input variables according to the three different conditions are given in Table 2.



◀ Figure 6: Input variables under different design conditions on a user-interface of Hermes.

5.2. Constraints

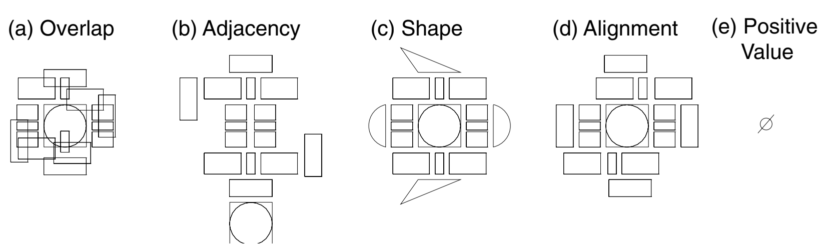
There are five basic constraints necessary for regulating the three prototypes of Villa Rotonda in hard, medium, and soft design conditions: Overlap constraints, Adjacency constraints, Shape constraints, Alignment constraints and Positive value constraints. Overlap constraints ensure that design components do not occupy the same area. Adjacency constraints regulate the laws of position among the components. Positive value constraints avoid any negative numeric values of the input variables that would cause an empty shape. All the design components are forced to be aligned with each other according to the design of the original villa. Shape constraints ensure that every design component keeps its designated formal entity derived from the original villa. Some plans that violate the basic constraints are given in Figure 7 as examples.

According to the constraints, the parametric values of input variables are adjusted and rearranged. For example, in a soft condition, when the value of design variable x_1 (*DCH*: diameter of central hall) is smaller than the sum of two x_5 (*LSR*: length of small room), x_6 (*HWLR*: width of hallway left and right side) and two x_8 (*TW*: thickness of wall), this violates the overlap constraint and possibly other constraints, too. To resolve this conflict, the dimensions of the design variables are recalculated as in Figure 8.

► **Table 2: Input variable and design conditions.**

	Component	Input Variable
Hard Condition: 7 input variables	DCH = Diameter of Central Hall	x_1
	Proportionality	**Input Variable**
	The first term of Proportionality *(DCH, WLR) = HWRL*	*propo_i*
	The second term of Proportionality *(WLR, LSR) = LLR or WSR*	*propo_j*
	The second term of Proportionality *(WSR, HWLR) = DP*	*propo_k*
	The second term of Proportionality *(WLR, LLR) = HL*	*propo_p*
	The second term of Proportionality *(WSR, LSR) = HS*	*propo_q*
	The second term of Proportionality *(DP, LP) = HC*	*propo_r*
	Component	**Input Variable**
Medium Condition: 11 input variables	*DCH* = Diameter of Central Hall	x_1
	WLR = Width of Large Room	x_2
	LLR = Length of Large Room	x_3
	WSR = Width of Small Room	x_4
	LSR = Length of Small Room	x_5
	HWLR = Left & Right side Hallway Width	x_6
	HWFR = Front & Rear side Hallway Width	x_7
	DP = Depth of Portico	x_9
	Proportionality	**Input Variable**
	The second term of Proportionality *(WLR, LLR) = HL*	*propo_p*
	The second term of Proportionality *(WSR, LSR) = HS*	*propo_q*
	The second term of Proportionality *(DP, LP) = HC*	*propo_r*
	Component	**Input Variable**
Soft Condition: 14 input variables	*DCH* = Diameter of Central Hall	x_1
	WLR = Width of Large Room	x_2
	LLR = Length of Large Room	x_3
	WSR = Width of Small Room	x_4
	LSR = Length of Small Room	x_5
	HWLR = Left and Right side Hallway Width	x_6
	HWFR = Front and Rear side Hallway Width	x_7
	DP = Depth of Portico	x_9
	HL = Height of Large Room	x_{16}
	HS = Height of Small Room	x_{23}
	HB = Height of Base	x_{21}
	HU = Height of Upper Level	x_{22}
	HC = Height of Column	x_{20}
	HCH = Height of Central Hall	x_{15}

► Figure 7: Basic constraints violated.



```
checkLB = x1-(2*x5+x6+2*x8);
if(checkLB < 0)
    x1 = (2*x5+x6+2*x8)
else
    x1 = x1
end
```

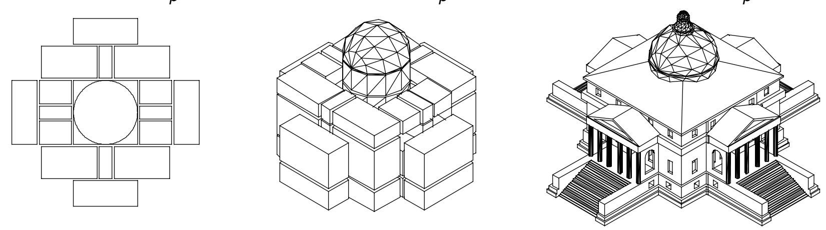
◀ Figure 8: A partial MATLAB code of conditional statement in mathematical relations.

5.3. Fitness functions

The fitness function of the proportionality synthesis on Villa Rotonda is to minimize $f(r) = V_r$ (remainder value) of the dimensions of the 32 design components with the computation of proportionality.

By minimizing $f(r)$, the maximization of proportionality value V_p of the 32 dimensions is achieved.

Computation of the fitness function starts with gathering the dimensions of the input variables. In the middle of the computation, the dimensions of necessary design variables according to the three prototypes are generated from the dimensions of the input variables. Using the dimensions from the input variables, three prototypes of the original villa are generated as below.



◀ Figure 9: Three prototypes of original Villa Rotonda.

Minimization of the remainder value $f(r)$ is subject to mathematical relations that represent the constraints under the design conditions: hard, medium, and soft. Some partial code of the mathematical relations under the hard design condition is given in Figure 10 as an example.

```
minimize f (r)

%with respect to input variables

%the range of input variables (design component)

minDCH <= x1 <= maxDCH

%the range of input variables (proportionality)

1<= the first term of propo_i <= 14
1<= the second term of propo_j <= 11
1<= the second term of propo_k <= 11
1<= the second term of propo_p<= 11
1<= the second term of propo_q <= 11
1<= the second term of propo_r <= 11
```

◀ Figure 10: A partial MATLAB code of the fitness functions for hard design condition.

%subject to design constraints

%maintain the original ratio $\frac{15}{13} = \sqrt{2}, \frac{30}{11} = \sqrt{3}$

x2= (13*x1)/15
x5= (11*x1)/30

%compute the value of a design component variable
with selected proportionality from proportionality variable

x6= the first term of propo_i (x2, x1)
tempx3 = the second term of propo_j (x5, x2)

tempx4 = x1/2
x3= select (tempx3, tempx4)
x4= select (tempx3, tempx4)
x9= the second term of propo_k (x6, x4)

x16= the second term of propo_p (x3, x2)
x23= the second term of propo_q (x5, x4)
x20= the second term of propo_r (x9, x1)

% the values of x1, x2, x5, x6, x3, x4, x9, x16, x23, and x20 are computed
% from one design component variable (x1) and six proportionality variables
% (propo_i, propo_j, propo_k, propo_p, propo_q, propo_r)
% the values of the other 22 variables are computed based upon the above

x15 = (11*x1)/6

x17 = (1.25*x16)/20.5
x18 = x17+x16

maxx20temp= x18-1.87
minx20temp= x18-7.5
minx20temp <= x20 <= maxx20temp

x19 = x18 - x20
x21 = (5*x20)/9
x22 = (7*x21)/10
x13 = x20/9
x8 = (x1 - (x6+2*x5))/2
x7 = (x1+2*x3)-(2*x2 + 2)
x11 = (x1+x8-5*x13)/4
x12 = (3*x11)/4
x14 = x1 - (x9+x13)
x10 = x1
x24 = x17
x25 = x17
x26 = x17/1.25
x27 = ((x8+8.5)*(x15-(x8+x22)))/26.25
x28 = x19/3
x29 = x19/6
x30 = x19/2
x31 = x30
x32 = x16 - (x26+ x23)

All the dimensions of the design component variables necessary for each prototype of Villa Rotonda are generated from a given range of input variables under a specific design condition. If a morphological structure established by the dimensions violates any constraints, then the associated set of the parametric values is omitted from the process of optimization by means of penalty functions.

5.4. Penalty functions

The intention of penalty functions is to avoid the computational loss of producing infeasible and undesirable designs by the optimization process. Penalty functions impose heavy weight values on the input candidates that violate the constraints. If one of the candidate dimensions is negative, the set of candidates automatically gets a penalty of 100% Remainder value V_r, which means 0% proportionality value V_p. The set is thrown out of the optimization loop.

5.5. Optimization method

A genetic algorithm is employed in the combinatorial search involving the design optimization of the morphological variations of Palladio's Villa Rotonda. The size of the genetic algorithm search is defined by a population size in each generation and the number of generations. Any dimensions in the range of input variables are computed by a random selection method in a precision value set by the user. The search is controlled by the population size of the input dimensions, the probability of crossover and mutation, and the number of generations.

5.6. Output

The computations of proportionality for Villa Rotonda produce four different types of outputs: 1) a text file that shows remainder value V_r and the best set of the dimensions of design variables at each generation; 2) a visual representation based upon the optimized dimensions in the text file; 3) a screen capture of the input settings and its evolutionary optimization process on a user-interface of a computer-based proportionality synthesis application; and 4) an animation made of the series of visual representations of the best design at every generation. This animation format is applied only to the plan prototype. According to the user's choices, extensive speculations and elaborated articulations on the generated results can be made in the analysis component of Hermes implemented in AutoCAD. A text file outcome from a proportionality synthesis on the detailed model prototype of Villa Rotonda is illustrated in Figure 11.

► Figure 11: A text file outcome.

96.6154 **% remainder value V_r**
%the values of 14 input variables under soft design condition
53.0000 25.0000 15.0000 12.0000 15.0000 9.0000 10.0000
13.0000 20.0000 16.0000 11.0000 5.0000 18.0000 63.0000
% remainder value V_r on three prototypes and the sum of them
Plan: 80.0000 Mass: 82.1678 Detailed: 96.6154 Sum: 58.7832
% the optimized dimensions of the 32 design variables required for constructing a detailed model
[x1, x2, x3, x4, x5, x6, x7, x9, x16, x23, x21, x22, x20, x15, x10, x8, x13, x32, x18, x11, x12, x14, x19, x27, x24, x17, x26, x25, x28, x29, x30, x31]=
(list 53.0000 25.0000 15.0000 12.0000 15.0000 9.0000
10.0000 13.0000 20.0000 16.0000 11.0000 5.0000
18.0000 63.0000 53.0000 7.0000 2.0000 3.0244 21.2195
12.5000 9.3750 38.0000 3.2195 30.1143 1.2195 1.2195
0.9756 1.2195 1.0732 0.5366 1.6098 1.6098)

6. REVIEW OF THE OUTCOMES

From the search for an optimum design having the best proportionality values derived from parametric variations of Villa Rotonda, two different types of outcomes are generated. One is a set of dimensions for an optimized Rotonda that has a very limited morphological difference from Villa Rotonda. This provides a reference to compare Palladio's usage of original proportionality and dimensions, and the one generated by Hermes. The other is a set of the descendants of Villa Rotonda having various morphological differences from the original villa while maximizing their proportionality values. This allows the possibility for Hermes to provide various design synthesis outcomes within the given mathematical model of the original Villa Rotonda, and draws the limitations of such proportionality synthesis.

6.1. Optimized Villa Rotonda

In order to achieve a maximum proportional balance embedded in the original villa, Hermes performs targeted parametric variations while maintaining the formal structure of the villa. In this process, Palladio's proportional treatment of the original villa is compared to the generated variations. A 0% tolerance is set for the parametric variations generated under hard condition within Hermes.

Variation of Existing Proportionalities

Based upon the study of Palladio's design of Villa Rotonda, his intention to encode several proportionalities in the building is clear. Originally, three proportionality relationships are employed for defining the horizontal dimensions and the other three are used for the vertical dimensions as below.

Number	Proportionality	Design Variable
4	Proportionality relation among (*HWLR, WLR, DCH*)	*propo_i*
10	Proportionality relation among (*LSR, LLR or WSR, WLR*)	*propo_j*
5	Proportionality relation among (*HWLR, DP, WSR*)	*propo_k*
1	Proportionality relation among (*LLR, HL, WLR*)	*propo_p*
1	Proportionality relation among (*LSR, HS, WSR*)	*propo_q*
10	Proportionality relation among (*DP, HC, LP*)	*propo_r*

◀ **Table 3: Palladio's choice of six proportionalities in his Villa Rotonda.**

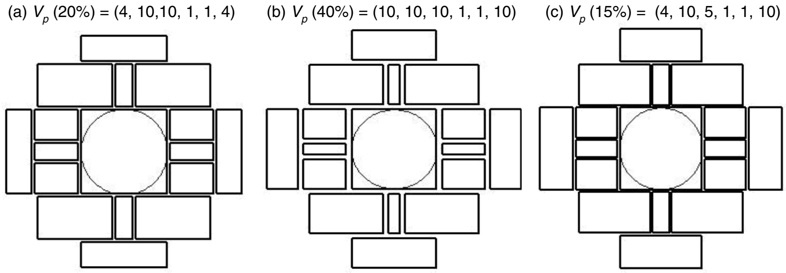
Then, the question is whether Palladio's selection of proportionality is the best choice according to proportionality value V_p when other design considerations are fixed. Under the hard design condition of the plan, mass model, and detailed model prototypes, the size of the diameter of the central hall and the numbers of six proportionalities are required as the input dimensions. When the size of the diameter of the central hall is fixed at 30, the original dimension of Villa Rotonda, all the possible choices of the six proportionalities are computed to find the best proportionality value V_p . Using the ranges of the six proportionalities, the search size for this investigation is calculated as 2,254,714 cases (14*11*11*11*11*11). Instead of checking this huge number of cases with a complete search, Genetic Algorithms (GA) is employed as an optimization method in order to find the best set of six proportionalities.

The initial conditions for performing the proportionality synthesis with GA are generation gap (0.9), precision (4), code (0), scale (0), number of populations (90), number of generations (100), probability of crossover (0.7), and probability of mutation (0.7). After review of 9000 cases from 100 generations with 90 populations, Hermes is used to find if Palladio's choice of the set of proportionality (*i, j, k, p, q, r*) = (4, 10, 5, 1, 1, 10) for Villa Rotonda makes the best proportionality values in three different prototypes under hard design condition. In the plan prototype, the proportionality value V_p of proportionality (*i, j, k, p, q, r*) = (10, 10, 10, 1, 1, 10) and (10, 10, 10, 1, 1, 4) is 40%, which is the first choice among the selections from the generations. A V_p of (4, 10, 10, 1, 1, 4) is 20% as the third. The V_p of (4, 10, 5, 1, 1, 10), the original proportionality set, is 15 % and is the eighth choice among the selections.

V_p (%)	*propo_i*	*propo_j*	*propo_k*	*propo_p*	*propo_q*	*propo_r*
40	10	10	10	1	1	10
40	10	10	10	1	1	4
20	4	10	10	1	1	4
20	10	10	5	1	1	4
20	10	10	6	1	4	6
20	10	10	6	4	4	6
20	4	10	10	2	5	5
15	**4**	**10**	**5**	**1**	**1**	**10**
15	4	10	5	1	2	10

◀ **Table 4: Proportionality value V_p with proportionality (i, j, k, p, q, r) in plan prototype.**

► Figure 15: Visual representations of (*i, j, k, p, q, r*) in plan prototype.

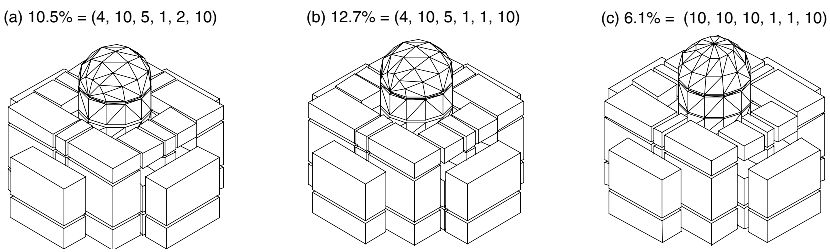


In the mass model prototype, the original set of proportionality (4, 10, 5, 1, 1, 10) is the first choice with 12.7%. The set (4, 10, 5, 1, 2, 10) is the second choice with 10.5%. Finally, the set (10, 10, 10, 1, 1, 10) is the third choice with 6.1% even though it was the first in the plan prototype.

► Table 5: Proportionality value Vp with (*i, j, k, p, q, r*) in mass model prototype.

V_p (%)	*propo_i*	*propo_j*	*propo_k*	*propo_p*	*propo_q*	*propo_r*
12.7	**4**	**10**	**5**	**1**	**1**	**10**
10.5	4	10	5	1	2	10
6.1	10	10	10	1	1	10
4.2	10	10	10	1	1	4
3.6	4	10	10	1	1	4

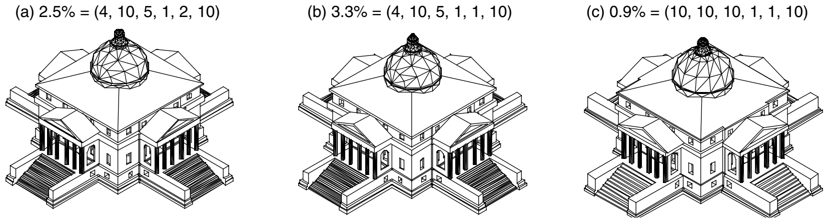
► Figure 16: Visual representations of (*i, j, k, p, q, r*) in mass model prototype.



In the detailed model prototype, the original set of proportionality (4, 10, 5, 1, 1, 10) is the first choice with 3.3%. The set (4, 10, 5, 1, 2, 10) is the second choice with 2.5%. Finally, the set (10, 10, 10, 1, 1, 10) is the third choice with 0.9 %.

► Table 6: Proportionality value V_p with (*i, j, k, p, q, r*) in detailed model prototype.

V_p (%)	*propo_i*	*propo_j*	*propo_k*	*propo_p*	*propo_q*	*propo_r*
3.3	**4**	**10**	**5**	**1**	**1**	**10**
2.5	4	10	5	1	2	10
0.9	10	10	10	1	1	10
0.6	10	10	10	1	1	4
0.5	10	10	5	1	1	4



◀ Figure 17: Visual representations of (*i, j, k, p, q, r*) in detailed model prototype.

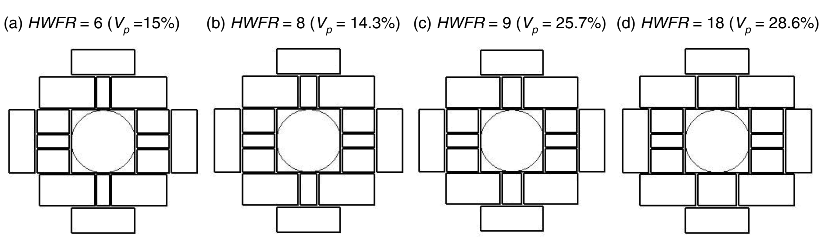
From the outcomes of the proportionality synthesis focusing on proportionality variation, we find that the set of proportionality (4, 10, 5, 1, 1, 10), originally chosen by Palladio about 450 years ago, places eighth in the plan prototype, but the first in the mass model and detailed model prototypes, those including vertical dimensions. It confirms that Palladio's "systematic linking" (Wittkower, 1971) and the notion of "beauty" (Tavernor and Schofield, 1997) employed in Villa Rotonda (Howard and Longair, 1982) provide the best design solution for maximizing proportional balance embedded in the design of the villa.

Variation of Width of Front and Rear Hallway

On the plate of Villa Rotonda in the second book of *Quattro Libri dell'Architettura*, the width of the left and right hallway (*HWLR*) is given as 6 Vincetine feet. However, the width of the front and rear hallway was not clearly indicated. It is assumed that the width is similar to that of the left and right hallway, which is 6. In terms of proportionality, the best value of *HWFR* is sought in the range from 1 to 30 feet under a soft design condition. When *HWFR* is set to 6 feet, its plan has the fifth value with 15%. When the width is set to 9, its plan has the second value with 25.7%, mass model has the best with 15%, and detailed model has the second with 3.7%. When the width is set to 8, its detailed model has the best value with 3.8%.

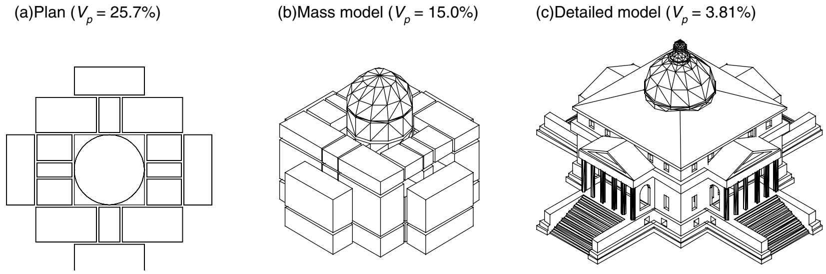
Plan		Mass Model		Detailed Model	
HWFR	**V_p (%)**	***HWFR***	**V_p (%)**	***HWFR***	**V_p (%)**
18	28.6	**9**	15.0	**8**	3.8
9	25.7	20	14.3	**9**	3.7
10	22.9	**8**	13.3	14	3.6
19	17.1	**6**	12.7	18	3.3
6	15	14	12.6	10	3.3
8	14.3	16	12.6	**6**	3.3

◀ **Table 7: Variations of *HWFR* under soft design condition.**



◀ Figure 18: Variations of *HWFR* in plan prototype.

► Figure 19: Three prototypes with *HWFR* = 9.

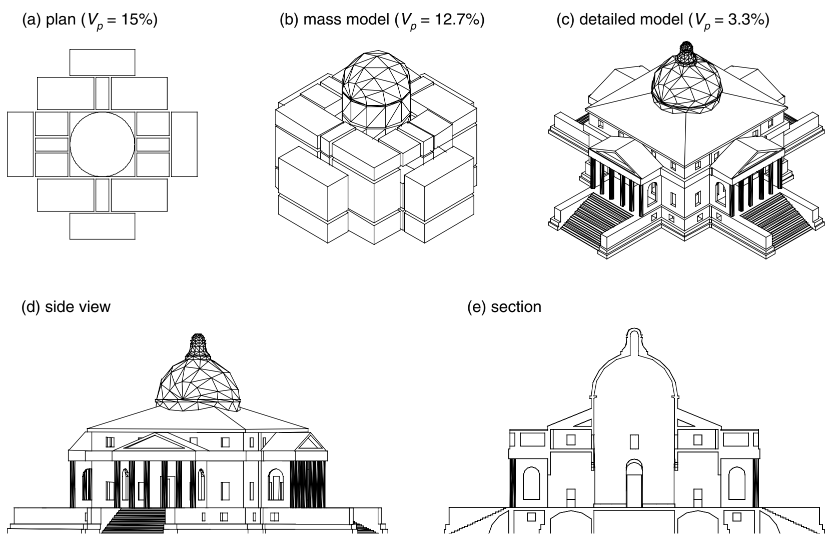


When 6 feet is taken as the width of *HWFR*, for each prototype the proportionality value is not as high compared to that provided by other candidates. Instead, when 8 feet or 9 feet are taken for *HWFR*, each prototype has a much higher proportionality presence, as shown in Table 7. Considering the proportionality values of 9 feet *HWFR* in plan, mass, and detailed model, we believe that 9 feet is a strong candidate for the missing dimension of the width of front and rear hallways of the original villa in terms of proportionality.

6.2. Descendants of Villa Rotonda

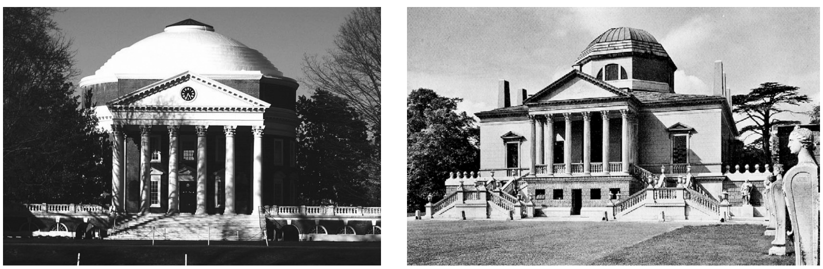
In this section, Hermes generates various morphological transformations from the original villa while maximizing their proportionality values. Villa Rotonda becomes an ancestor of a family that consists of close relatives, medium relatives and distant relatives. A 0% tolerance is set for these investigations.

► Figure 20: Palladio's Villa Rotonda.



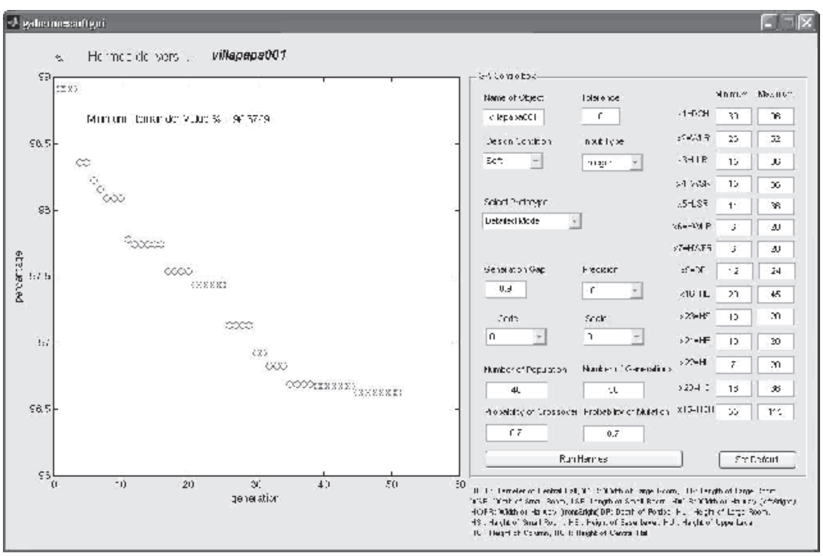
There are seven procedures for making the descendants of Villa Rotonda under the soft design condition within Hermes: 1) establishing a basic concept of design, 2) defining the ranges of the dimensions of input variables according to the concept within the receptors of the synthesis component of Hermes in MATLAB, 3) performing the optimization of the dimensions with Hermes in MATLAB, 4) visualizing the alternative dimensions provided from MATLAB with Hermes in AutoCAD, 5) selecting the optimum among the visualized alternatives, 6) adjusting the optimum by narrowing down the ranges of the input dimensions, and 7) continuing the procedures until the best is found both in consideration of Hermes's proportionality value and the user's design concept. Through these procedures, three different variations of Villa Rotonda were generated: Villa Papalambros, Villa Economou, and Villa Kim. The concept of Villa Papalambros is to emphasize the idea of unity embedded in the central hall of the original Villa Rotonda (March, 1998) by increasing its diameter and height and regulating other components to support the central hall. Villa Papalambros is designed after Thomas Jefferson's Rotunda (1826) and Lord Burlington's Chiswick House (1725). Jefferson's Rotunda is the result of his interpretation of the Pantheon in Rome. Lord Burlington's Chiswick House is an example of an English Palladian villa, which highlights the central hall (Tavernor, 1991).

(a) The Rotunda (from Univ. of Virginia) (b) Chiswick House (from Tavernor, 1991)



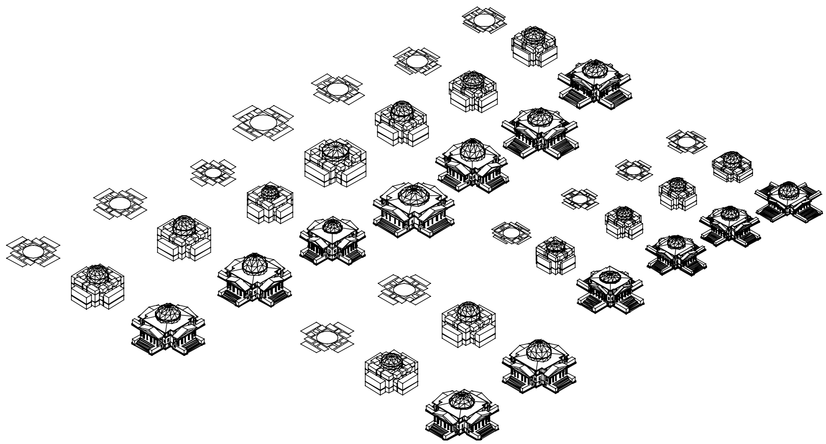
◀ Figure 21: Jefferson's Rotunda and Lord Burlington's Chiswick House.

According to the concept, the ranges of the initial inputs are defined as below (Figure 22).



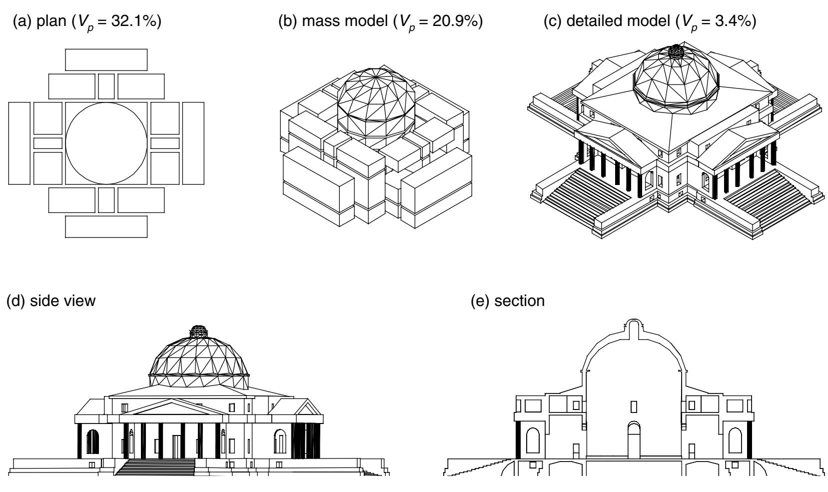
◀ Figure 22: The optimization process of Villa Papalambros.

Using optimized dimensions, alternatives of Villa Papalambros are visualized in AutoCAD, in Figure 23. Then, the optimum design is selected from among them.



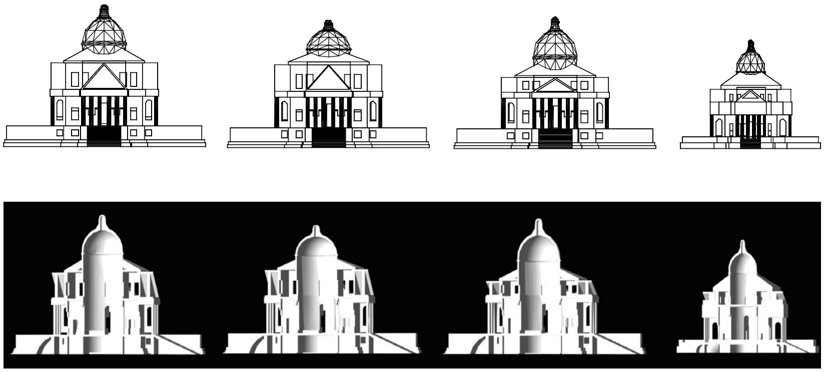
► Figure 23: The alternatives of Villa Papalambros.

Through the process of adjusting the dimensions of the selected villa, the final version of Villa Papalambros is achieved in Figure 24.

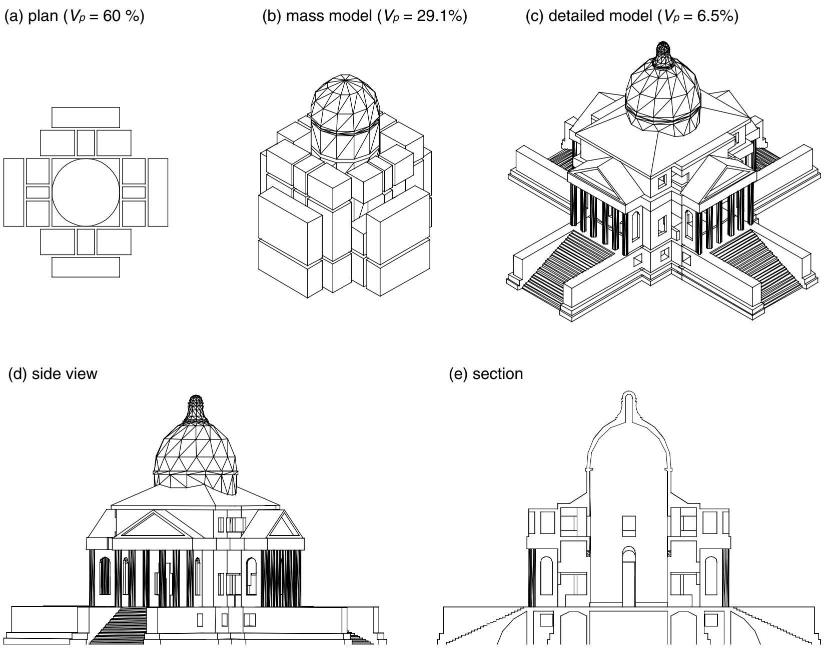


► Figure 24: Villa Papalambros.

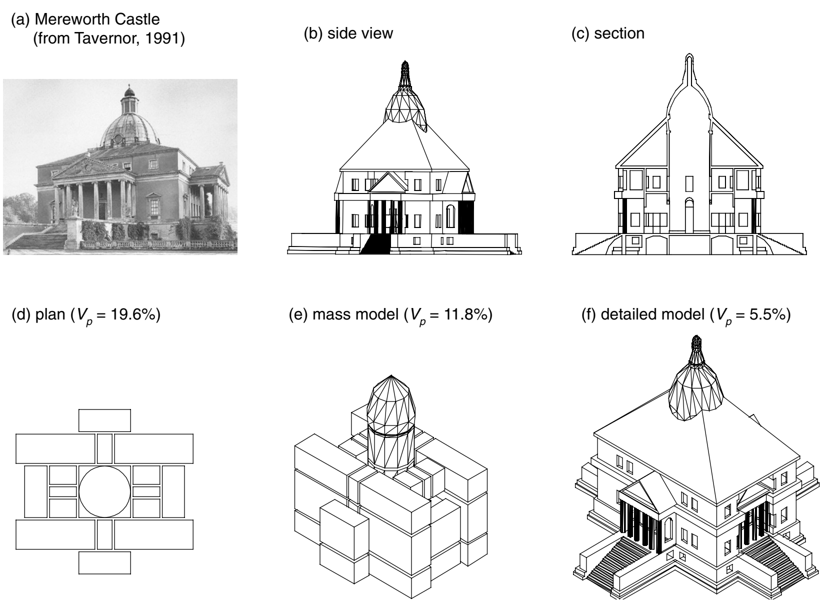
The concept of Villa Economou is achieved by highlighting its vertical components while maximizing the presence of proportionality value in the villa. The vertical components are the height of the central hall, the heights of each of the different levels including base level, entrance level, upper level, and the height of the dome. Among several alternatives, the tallest was selected for Villa Economou, as illustrated in Figure 26.



◀ Figure 25: The alternatives of Villa Economou.



◀ Figure 26: Villa Economou.



◀ Figure 27: Villa Kim.

Villa Kim is designed after Campbell's Mereworth Castle and then developed to introduce a new generation of Villa Rotonda as illustrated in Figure 27.

The dimensions of the primary design components of the three villas are compared to Palladio's Villa Rotonda, as seen in Table 8.

► Table 8: Numerical comparisons between the variations and the original villa.

	Villa Rotonda	Villa Papalambros	Villa Economou	Villa Kim
DCH	30	92	48	42
WLR	26	52	24	64
LLR	15	28	18	24
WSR	15	32	16	22
LSR	11	36	18	15
HWLR	6	12	8	8
HWFR	6	18	12	12
DP	12	24	14	18
HL	20.5	42	41	41
HS	13	20	18	32
HB	10	18	24	22
HU	7	16	32	24
HC	18	36	36	36
HCH	55	112	124	152
Plan (V_p %)	15	32.1	60	19.6
Mass (V_p %)	12.7	20.9	29.1	11.8
Detail (V_p %)	3.3	3.4	6.5	5.5

7. CONCLUSION

The process of learning a formal composition in architecture and refining it for a better design result usually involves meticulous trial and error routines, which require a considerable amount of patience and effort. The computational power of Hermes minimizes the mathematical sophistication demanded for the study of proportional theory in architecture. In providing an interactive user-interface and feedback system with various outputs in numerical and visual formats, including a rapid instantiation of three-dimensional geometric models, Hermes allows a designer to understand a master's original manipulation of proportion in a given design artifact with "reflection in action" (Shön, 1987). In this case, Genetic Algorithm (GA) is employed for 1) performing targeted parametric variations of the original villa in order to find an optimum villa design based upon proportional balance while maintaining the formal structure of the original villa, and 2) generating various morphological transformations from the original villa while allowing a user's interaction to guide the transformations (Michalek, 2001; Kelly et al, 2006). In the process of finding the optimized villa, Hermes provided numeric data that confirms the excellence of Palladio's treatment of proportional balance in the design of Villa Rotonda (Wittkower, 1971;

Howard and Longair, 1982; Tavernor and Schofield, 1997). According to the computation of proportionality in the design of Villa Rotonda, 8 or 9 Vicentine feet, instead of the 6 feet used for the width of the left and right hallway, are proposed as the candidates for the width of the front and rear hallway of the original villa, a dimension not indicated in Palladio's second book. The above examples demonstrate Hermes's value as a potential analytic tool for the restoration of an architectural precedent having very limited references. Furthermore, in exploring the morphological descendants of Villa Rotonda, we found that even though the descendants inherited the design character of Villa Rotonda, they exhibited their own unique identities according to the proportional balance embedded in their own formal structure. This suggests Hermes as a synthetic tool for understanding architectural style, in this case, the Palladian style, with respect to proportional balance.

In future researches, the addition of generative functions allowing various combinations of design rules to the current version of Hermes will provide a higher degree of freedom in the design of the Palladian style and the investigation of its extremes. Modification of Hermes' mathematical models according to the design rules of any given precedents can extend the application of Hermes for the formal study of various types and periods of design artifacts, not limited to architectural design. Furthermore, upon including other fitness criteria like structure, function, and cost to its mathematical models, Hermes can evolve from a formal design study tool to a comprehensive design decision-making tool.

References

1. Bentley, P., *Evolutionary Design by Computers,* Morgan Kaufman Inc., San Francisco, 1999.
2. Broughton, T. A., and Coates, P. S., The Use of Genetic Programming in Exploring 3d Design Worlds, in: Junge, R., eds., *CAADFutures,* Kluwer, Dordrecht, 1997, 885–917.
3. Conway, W. M., *Literary Remains of Albrecht Dürer,* Cambridge University Press, London, 1889.
4. Goldberg, D. E., *Genetic Algorithm in Search, Optimization and Machine Learning*, Addison-Wesley Publishing Company Inc., New York, 1989.
5. Heath, T., *A History of Greek Mathematics*, Clarendon Press, Oxford, 1921.
6. Hersey, G., and Freedman, R., *Possible Palladian Villas (Plus a Few Instructively Impossible Ones)*, MIT Press, Cambridge, 1992.
7. Howard, D., and Longair, M., Harmonic Proportion and Palladio's Quattro Libri, *Journal of the Society of Architectural Historians,* 1982, 41, 116-143.
8. Jagielski, J., and Gero, J. S., A Genetic Programming Approach to the Space Layout Planning Problem, in: R. Junge, ed., *CAAD Futures,* Kluwer, Dordrecht, 1997, 875–884.
9. Kelly, J., Wakefield, G. H., and Papalambros, P. Y., The Development of a Tool for the Preference Assessment of the Visual Aesthetics of an Object Using Interactive Genetic Algorithms. *The proceedings of 9th Generative Art Conference,* Milan, Italy, December 13–15, 2006.

10. Krier, R., Chapter IV On Proportions, in *Architectural Composition*, Rizzoli, New York, 1988.
11. March, L., *Architectonics of Humanism: Essays on Number in Architecture*, Academy Editions and John Wiley, London, 1998.
12. Michalek, J. J., *Interactive Layout Design Optimization*, Master of Science Thesis, the University of Michigan, 2001.
13. Mitrovic, B., Palladio's Theory of Proportion and the Second Book of the Quattro Libri dell' Architettura, *Journal of the Society of Architectural Historians*, 1990, 49, 279–292.
14. Papalambros, P.Y., and Wilde, D. J., *Principles of Optimal Design, Modeling and Computation*, Cambridge University Press, New York, 2000.
15. Park, H. J., *A Quantification of Proportionality Aesthetics in Morphological Design*, Doctoral Dissertation, the University of Michigan, 2005.
16. Schön, D. A., *Educating the Reflective Practitioner,* Jossey-Bass, San Francisco, 1987.
17. Semenzato, C., *The Rotonda of Andrea Palladio,* (trans. Percy, A.), The Pennsylvania State University Press, University Park & London, 1970.
18. Shea, K., and Cagan, J., Innovative dome design: applying geodesic patterns with shape annealing, *Artificial Intelligence for Engineering Design, Analysis, and Manufacturing,* 1997, 11, 379–394.
19. Steadman, P., *The Evolution of Designs,* Cambridge University Press, New York, 1976.
20. Tavernor, R., *Palladio and Palladianism,* Thames & Hudson Ltd., Cambridge, 1991.
21. Tavernor, R., and Schofield, R., *Andrea Palladio: Four Books on Architecture*, MIT press, Cambridge, 1997.
22. Testa, P., O'Reilly, U.M., Greenwold, S., and Hemberg, M., AGENCY GP: Agent-Based Genetic Programming for Spatial Exploration, in: Goodman, E. D., eds., *2001 Generative Evolutionary Computer Conference*, San Francisco, 2001.
23. Thompson, D.W., *On Growth and Form*, Cambridge University Press, Cambridge, 1961.

Hyoung-June Park

University of Hawaii at Manoa
School of Architecture
2410 Campus Rd., Room 301H, Honolulu, HI 96822

hjpark@hawaii.edu