STYLISTIC REPRODUCTIONS OF MONDRIAN'S COMPOSITION WITH RED, YELLOW, AND BLUE

HYOUNG-JUNE PARK
[1]*School of Architecture, Univesrity of Hawaii at Manoa*
[1]*hjpark@hawaii.edu*

Abstract. Shape grammars are employed for analyzing and delineating the formal structure of Mondrian's painting. The proportionality (dynamic equilibrium or commensurability) embedded in the structure of the artifact is optimized with Genetic Algorithms. The optimization process introduced in this paper allows a user's intervention to provide a guided search for finding stylistic reproductions of the original. Two types of the stylistic reproductions are conducted: 1) generating formal descendants of the original, and 2) tuning the original structure. The implementation of the reproductions is described also.

Keywords. Mondrian; Style; Shape Grammar; Proportionality; Genetic Algorithms.

1. Introduction

The motivation for generating stylistic reproductions conducted in this paper comes from the design of a building façade delineating proportional patterns embedded in Mondrian's "Composition with Red, Yellow, and Blue." His painting is known to be the artifact of delivering "living rhythm," which is the essence of dynamic equilibrium created from proportions. In his "Plastic Art and Pure Plastic Art," Mondrian explains that the development of the dynamic equilibrium guides the development of the laws that consist of the relationships of position and dimension of the constructive elements in the painting.

Shape Grammars have been applied for the analysis and synthesis of various artefacts including buildings, artworks, industrial products, and historical contents. The set of shape rules with the basic elements and their transformational relationships represent the morphological structure of the given artefact in shape grammars (Cenani and Cagdas, 2007; Knight, 1980). Furthermore, shape grammars have been employed for understanding design styles and languages (Koning and Eizenberg, 1981; Li, 2004; McCormack et al, 2004). Shape Grammars represent style and explain the purpose of style characterization 1) with the clarification of the underlying commonality embed in a given artefact; (2) with providing a prototype or a yardstick for the style; (3) as a compositional machinery for new instances of the style (Stiny and Mitchell, 1978); and (4) with its descriptive power to reveal intrinsic simplicity or regularities in designs (Park, 2008). With maintaining an embedded style, shape grammars allow a designer

RE: Anthropocene, Proceedings of the 25th International Conference of the Association for Computer-Aided Architectural Design Research in Asia (CAADRIA) 2020, Paper 25 (Preprint).

to generate infinite variations of an artefact by changing parametric values of the morphological structure of the artefact.

Based upon proportionality (March,1998; Park, 2017) embedded in the painting, a computational application for generating stylistic reproductions of Mondrian's "Composition with Red, Yellow, and Blue" is introduced in this paper. Proportionality is a commensurability (dynamic equilibrium) providing proportional balance among three different terms. With the analysis of the morphological structure of Mondrian's painting, the series of shape rules, schema, and the labeled parameters are defined to delineate the re-productions of the original. The rules and parameters become input variables, constraints, and objective functions of the set of mathematical models. The models are developed for the optimization of the proportionality embedded in the formal structure of the painting. Genetic Algorithms are employed for this guided optimization in order to generate the stylistic reproductions with maximizing the proportionality (kelly et al, 2010). The proposed application including its input, output system, and user-interface is implemented within MATLAB, a multi-paradigm numerical computing environment and proprietary programming language developed by MathWorks.

2. Design Schema and Variables

Mondrian's "Composition Red, Yellow, and Blue" painted in 1930 with oil on canvas has 45 cm x 45cm dimensions. It has been exhibited in Kunsthaus, Zurich.

Figure 1. A scanned image of "Composition with Red, Yellow, and Blue" (Blot-kamp, 1994).

The geometric model of the scanned raster image is constructed in a vector graphic within AutoCAD. The position of lines, constructive elements or basic elements, is in rectangular form. The dimension of the lines is defined with a scale of 45 cm. The dimensions of 36-line segments are identified as essential design components of the painting. The design schema of the painting is illustrated in Figure 2. The decomposition of the original structure with the schema becomes the basis of defining the geometric relationship in order to perform the parametric variations of the original.

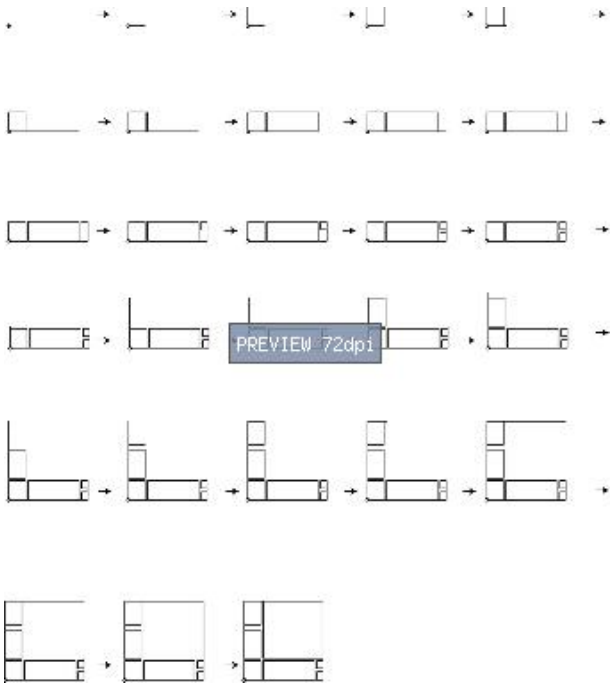

Figure 2. Design schema of the formal structure .

With the schema, the 36 line segments are defined as design variables to have various parametric values as $x_i, i = 1, \ldots, 36$. The reconstructed geometric representation of the painting is illustrated with the dimensions in Figure 3.

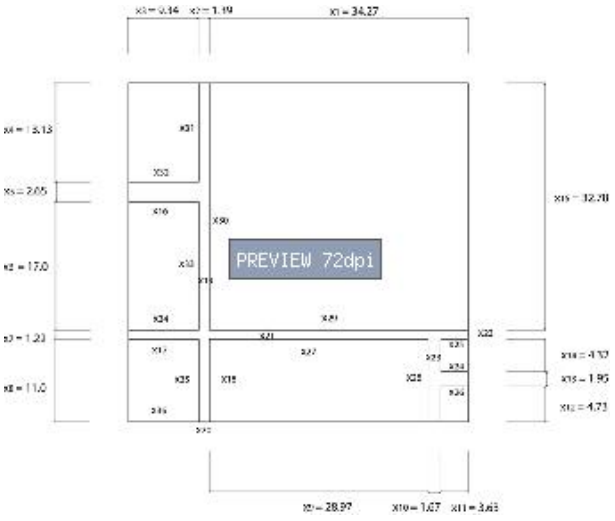

Figure 3. The geometric representation of 36 variables & original dimensions.

3. Objective Functions and Criteria: Proportionality

The relationships between the parts and the whole are treated in mathematics as commensurability. Commensurability among three ordered terms and their differences is established when they have equality among their proportional relationships. When the commensurability is achieved among them, we achieve a proportional equilibrium among the parts and the whole. There are 11 ways of representing three ordered numbers and their differences with the equalities of the ratios among them, which were developed by ancient Greek mathematicians (Heath, 1921; D'ooge, 1926; March, 1998). The 11 ways of representing the equalities of the ratios among the three ordered numbers and their differences are defined as "Proportionality" (March, 1998, Park, 2017).

The goal of the optimization is to maximize proportionality (dynamic equilibrium) value, the essence of Mondrian's style, embedded in the painting. The proportionality value from the combination of 36 variables is employed as a fitness criterion for the optimization of the artifact. The objective function of the proportionality synthesis on the painting is to minimize $f_r = V_r$ (remainder value) of the dimensions from the 36 design variables (n = 36) with the computation of proportionality. By minimizing f_r subject to the values of the input variables, the maximization of the proportionality value V_p of the design variables is achieved. The computation of the objective functions starts with the dimensions of the input variables. The dimensions of the 36 design variables are generated from the dimensions of the input variables. Results for all triplet combinations for the 36 variables are aggregated statistically and a final value, the proportionality value V_p, is computed as a percent of the number of triplets observing any form of proportionality relative to the total number of the triplet combinations. The computation of individual proportionality value P_k, proportionality value V_p, and remainder value V_r are illustrated as below.

k proportionality number (1˜11)

n number of the dimensions of design variables

x_i i^{th} parametric value (dimension) of design variable, $i = 1, \ldots, n$

L the list of triplet combinations from x_n

A_k an algorithm computing number of triplet combinations within P_k

$L_k = A_k(x_n)$ the number of triplets by algorithm A_k to x_n in P_k

$T_k = |L_k|$ the number of triplets in the list L_k

T the total number of triplets combinations from n

The total number $T = C(n, 3) = \dfrac{n!}{(3!(n-3)!)}$

Individual proportionality values $P_k = \dfrac{100 \cdot T_k}{T}$

Proportionality value $V_p = \sum_{k=1}^{K} P_k$

Remainder value $V_r = 100 - V_p$

4. Constraints

From the computation of the objective functions, the dimensions of the 36 design variables are generated for a given range of the input variables. If the morphological structure established by the dimensions violates any given constraints, then the set of the dimensions computed from the input range is not passed on to the optimization process by penalty functions. There are five basic constraints necessary for maintaining the essence of the design of Mondrian's painting: Overlap, Adjacency, Rectangular shape, Boundary configuration, and Positive value of V_p. The visual representations of the cases that violate the basic constraints are given below.

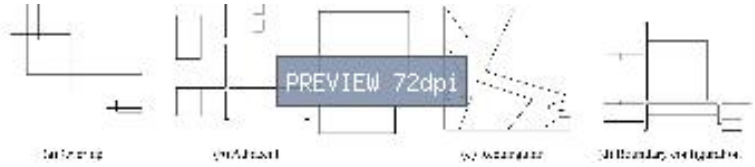

Figure 4. Design Constraints.

5. User Interface: Receptors and Effectors

The receptor and effector are the user-interface to perform a guided search for a formal structure of Mondrian's "Composition with Red, Yellow, and Blue" with an optimal proportionality value. The receptor includes design definition, genetic algorithm setting, input variables, and reset & execute. The effector includes output (plot of the optimization process and morphological transformations). There are three stages of an optimization process on the user-interface: 1) initial stage, 2) evaluation stage, and 3) representation stage. At the initial stage, the application provides the original parametric values of Mondrian's painting and its morphological structure as in Figure 5.

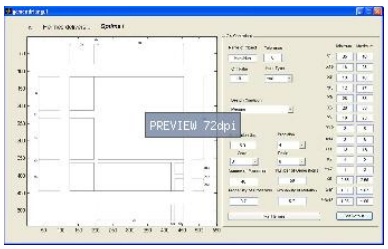

Figure 5. Initial stage .

According to a user's input, the optimization proceeds with minimizing the remainder value V_r for each generation. The trace of this evolution is recorded at the end of the evaluation stage. The visual representation of the output is made at the representation stage.

Figure 6. a) Evaluation stage b) Representation stage .

6. Output

The output from the design optimization conducted with proportionality synthesis on the painting of Mondrian has four different output formats: 1) a text file that shows remainder value V_r, and the best set of dimensions of design variables at each generation; 2) a visual representation based upon the optimized dimension in two different formats (line drawing and color-filled drawing); 3) a screen capture of the input settings and its evolutionary optimization process on a user-interface of a computer-based proportionality synthesis application; and 4) an animation made of the series of visual representations of the best design for every generation.

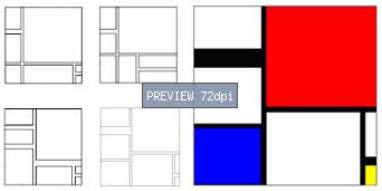

Figure 7. Output samples (a)Line (b)Color -filled.

7. Stylistic Reproductions

The various morphological transformations with maximizing the proportionality values (V_p) embedded in the formal structure of the original painting have been generated. The first approach is to generate the stylistic reproductions of the original painting, as its morphological descendants, by performing a guided optimization. The ranges of the dimensions of input variables are defined according to a user's concept within the receptors of the synthesis component in MATLAB. Accordingly, the animation of the alternatives is reviewed to adjust the optimum with narrowing down the ranges of the input dimensions. With continuing the procedures, four stylistic reproductions of the original were generated in the consideration of the proportionality value of the design and a user's design concept. For the comparison of the stylistic reproductions to Mondrian's other paintings, Figure 9 shows similar Mondrian's paintings at different times.

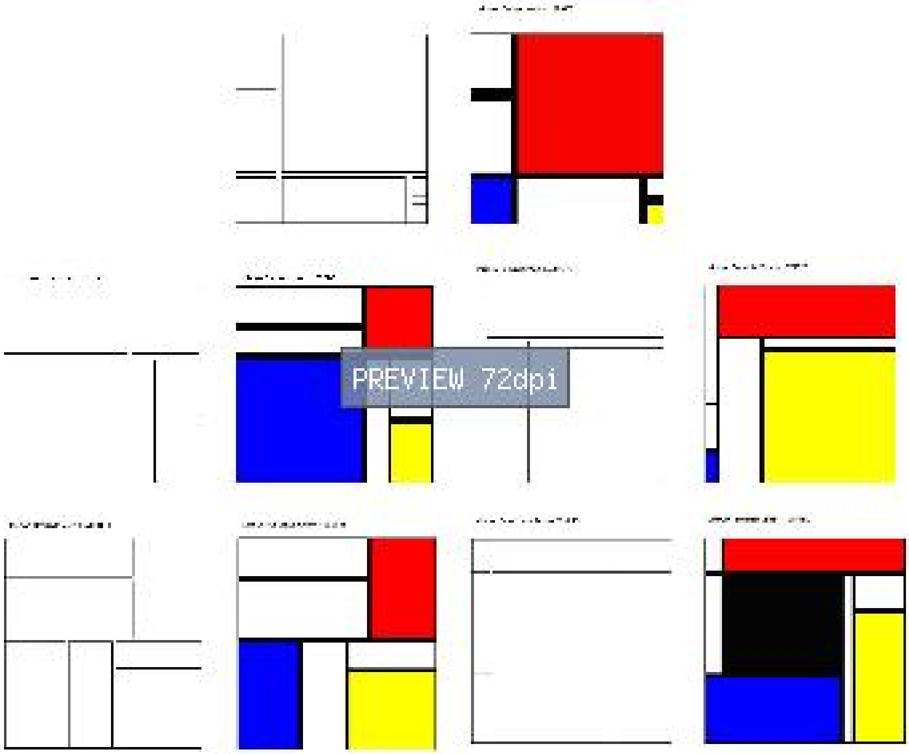

Figure 8. Four stylistic reproductions of the original.

Table 1. Numerical comparisons (R: Red, B:Blue, Y:Yellow, K: Black).

	Original Composition	Composition RBY	Composition YRB	Composition YBK	Composition KBRY
[illegible]	[illegible]	24	48	18	44
[illegible]	32.78	24	[illegible]	36	8
[illegible]	11	45	0	22	16
[illegible]	[illegible]	45	0	26	4
[illegible]	28.07	8	11	12	2
[illegible]	17	8	12	16	24
[illegible]	13.13	[illegible]	32	[illegible]	8
[illegible]	4.75	[illegible]	[illegible]	18	4
[illegible]	4.32	[illegible]	[illegible]	4	28
[illegible]	3.53	15	36	24	4
[illegible]	1.09	2	[illegible]	[illegible]	12
[illegible]	[illegible]	2	[illegible]	[illegible]	1
[illegible]	2.85	3	[illegible]	[illegible]	1
[illegible]	1.57	1	[illegible]	[illegible]	1
[illegible]	1.95	0	[illegible]	[illegible]	1
[illegible]	[illegible]	[illegible]	12.7	12.8	17.8
[illegible]	69.3	66.7	87.3	82.4	82.4

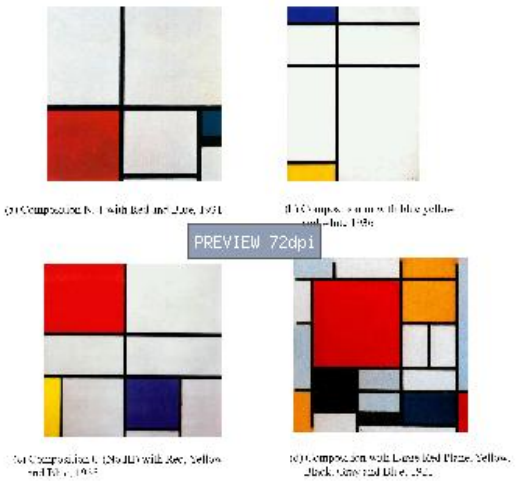

Figure 9. Other Mondrian paintings created in 1931, 1936, 1935, and 1921 .

The other approach is to tune the structure of the original painting with maximizing proportionality value. Through the tuning, the proportional value went up to 18.2% even with 0% tolerance.

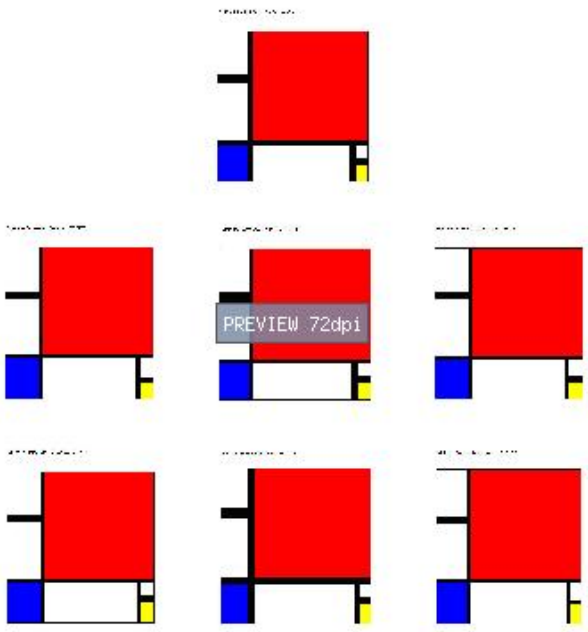

Figure 10. Tuning of the original.

8. Discussion

In this paper, the stylistic reproductions of Mondrian's "Composition with Red, Yellow, and Blue" were generated by optimizing proportional balance as a living rhythm or dynamic equilibrium embedded in the morphological structure of the painting. The design schema of the original painting became the instrument for 1) defining the geometric relationships of the basic elements in the original and 2) performing its parametric variations. With Genetic Algorithms (GAs), a guided search through a user's intervention within the interface of the proposed application is applied for the optimization (Kelly et al, 2010).

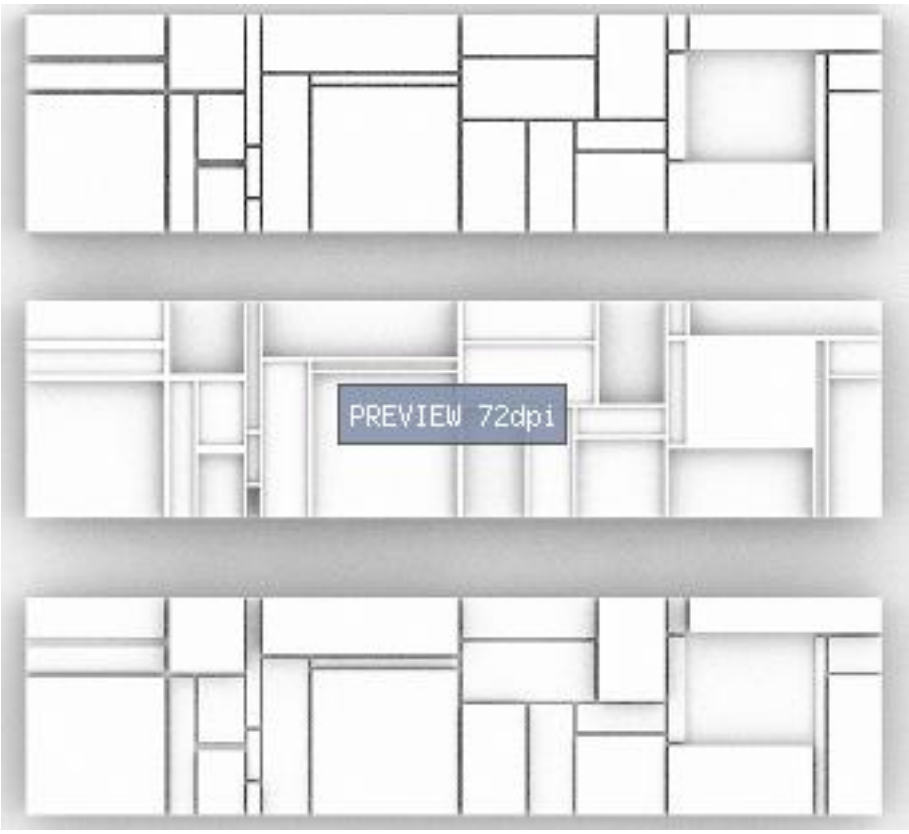

Figure 11. Design alternatives of a building facade.

As the outcomes, two types of the stylistic reproductions were presented. The morphological descendants of the original exhibited their own characters with maintaining a higher level of proportional balance than the one of the original. At the same time, they showed their stylistic resemblance to Mondrian's paintings in the contemporary years of Mondrian's "Composition with Red, Yellow, and Blue." The descendants have been applied for the motif of a façade design as shown in Figure 11. The facade design is introduced as an architectural application and

came from differentiating the heights of rectangular partitions of the descendants (Schnier and Gero, 1998). The proposed approach of the stylistic reproductions will be further developed in the design of a building layout (Michalek et al, 2002).

With the other type of reproductions, the higher quality of the proportional balance of the painting was tuned and restored. The proportionality values of the tuned paintings went up to $V_p = 18.2$ with 0% tolerance is higher than the one of the original painting, $V_p = 12.5$ even with 2.9% tolerance that is the range of Just Noticeable Difference (Fechner, 1966) in Vision. It may lead to a new definition of the original painting of Mondrian: "what is the original?"

References

Blotkamp, C.: 1995, *Mondrian: The Art of Destruction*, H.N. Abrams, New York.

Cenani, S. and Cagdas, G.: 2007, A shape grammar study: form generation with geometric Islamic patterns, *Proceedings of the 10th Generative Art Conference*, Milan.

D'Ooge, M.L.: 1938, *Nichomachus of Gerasa: Introduction to Arthmetic*, University of Michigan, Ann Arbor.

Fechner, G.: 1966, *Elements of Psychophysics*, Holt, Rinehart and Winston, Inc, New York.

Heath, T.: 1921, *A History of Greek Mathematics*, Clarendon Press, Oxford.

Kelly, J.C., Wakefield, G.H. and Papalambros, P.Y.: 2010, Evidence for Using Interactive Genetic Algorithms in Shape Preference Assessment, *International Journal of Product Development*, **13**(2), 168-184.

Knight, T.: 1980, The generation of Hepplewhite-style chair-back designs, *Environment and Planning B*, **7**, 227–238.

Koning, H. and Eizenberg, J.: 1981, The language of the prairie: Frank Lloyd Wright's prairie houses, *Environment and Planning B*, **8**, 295–323.

Li, A.: 2004, Styles, grammars, authors, and users, *Design Computing and Cognition '04*, 197–215.

March, L.: 1998, *Architectonics of Humanism: Essays on Number in Architecture.*, Academy Editions, Los Angeles.

McCormack, J.P., Cagan, J. and Vogel, C.M.: 2004, Speaking the Buick language: capturing, understanding, and exploring brand identity with shape grammars, *Design Studies*, **25**, 1-29.

Michalek, J.J., Choudhary, R. and Papalambros, P.Y.: 2002, Architectural Layout Design Optimization, *Engineering Optimization*, **34**, 461-484.

Mondrian, P.: 1937, *Plastic Art and Pure Plastic Art*, Witternborn, Schultz, Inc, New York.

Park, H.J.: 2008, Parametric Variations of Palladio's Villa Rotonda, *International Journal of Architectural Computing (IJAC)*, **6**, 145-169.

Park, H.J.: 2017, Ratios from the Intersections of 10 + 1 Proportionalities, *Nexus Network Journal*, **19**, 145 – 154.

Schnier, T. and Gero, J.S. 1998, From Mondrian to Frank Lloyd Wright: Transforming Evolving Representations, *in* I.C. Parmee (ed.), *Adaptive Computing in Design and Manufacture*, Springer, London, 207-219.

Stiny, G. and Mitchell, W. J.: 1978, The Palladian grammar, *Environment and Planning B*, **5**, 5-18.